Response to reviewers for manuscript AMT-2022-74: **Average visibility that has been miscalculated**

We appreciate the editorial team and the reviewers for their time and comments towards improving our manuscript. We have revised the manuscript carefully, and we respond to all of the reviewers' points below. Responses are given in red.

Anonymous Referee #1

General Comments:

The authors divided the concerns of the reviewers into three questions and answered them by means of some examples. However, the authors fail to improve the manuscript as suggested by the reviewers. The claim that the previous methods were wrong, is not satisfactory! Detailed comments on the authors' responses are below.

General Response: Many thanks to the reviewer for the helpful comments. This time we have made substantial changes to the manuscript to address the reviewers' comments. Specifically, we restrict our focus to proposing a new method for calculating average visibility, rather than dwelling on proving that the old method is incorrect and the new method is correct. We have revised the title, the structure, the argumentative process, and the conclusion of the manuscript. Due to the number of changes, we did not highlight the revised sentences with a different colour. Detailed responses to the comments are below.

1. Questions 1: The authors tried to explain through a car travel example that "is this a change from "incorrect" to "correct" or a "general improvement". I don't believe this is the case for visibility calculations and the example given nowhere proves that the previous visibility calculations were incorrect. How did the authors come to the conclusion that the all previous visibility calculations were similar to those in Equation 1 and their Equation 3?

Response: In accordance with the reviewer's comments, we no longer use the words "incorrect" and "correct" to describe the differences between the old and new calculation methods in the revised manuscript. Here, as a discussion, we explain our understanding of the difference between "a change from incorrect to correct" and "a general improvement", and how our thinking changed.

We think that if the previously used method contradicts basic scientific principles, and the new method corrects this contradiction, then it is "incorrect" to "correct". If the old method does not contradict basic scientific principles, and the new method leads to a better result, then it is a "general improvement".

The key question here is whether calculating the average of visibility data is a mathematical or physical problem. We have always considered it a physical problem. And we think a physical problem should have its physical meaning and corresponding process. We tried to use the car travel example in our last response to show that the previously used calculation method for average visibility was flawed in a physical sense, but we had not made that clear.

The previously used method carries out the summation over the visibility when calculating the average of visibility data. The visibility is neither an extensive quantity nor an intensity quantity, so that the results of the summation of visibility are just numerical values with no corresponding physical process. Therefore, the arithmetic average visibility calculated by the previously used method has no real physical meaning, leading to problems in the average extinction coefficient obtained using arithmetic average visibility. The revised manuscript elaborates on this point. On this basis, the new calculation method is justified from the point of view of physical processes, and the argument continues to explain why the harmonic average visibility calculated by the new method better reflects the changes in average atmospheric extinction coefficients and aerosol concentrations.

If the calculation of average visibility can only be understood as a physical process, then the previously used method is considered to be "incorrect" because of the progress without real physical meaning, and the proposed method is considered to be "incorrect" to "correct" compared to the old one. However, in retrospect, if the purpose is to numerically describe the measured visibility data, then the calculation of average visibility can be treated as a mathematical problem; therefore, it cannot be simply assumed that the previously used method is "incorrect".

Therefore, the expressions "correct" or "incorrect" are not used in the revised manuscript. We turn to emphasize that it is important to select the appropriate method for calculating the average visibility when the average visibility is used as a parameter to characterize changes in atmospheric extinction coefficients and aerosol concentrations.

2. Questions 2: I agree mathematically their example may be correct but does this example always apply in a real situation where atmospheric visibility cannot be infinite? Therefore, this example does not prove that the previous methods were wrong. I believe previous studies considered atmospheric conditions to calculate visibility and there is always room for improvement in the method depending on the problem and situation.

Response: We agree that an example can illustrate the existence of a problem, but it cannot be the basis for a conclusion. We revised the manuscript and give a theoretical derivation to obtain the conclusion. In brief, Eq. 1 gives the expression of the average extinction coefficient derived using previously used arithmetic average visibility, and the detailed derivation of which is given in the manuscript. The extinction coefficient is the product of the mass concentration and the mass extinction coefficient of the substance, as the definition implies. However, the expression of the extinction coefficient in Eq. 1 contradicts the definition of the extinction coefficient. Equation 2 gives the expression of the average extinction coefficient derived using the proposed harmonic average visibility, and the detailed derivation of which is given in the manuscript. The expression of the extinction coefficient in Eq. 2 is exactly in line with the definition of extinction coefficient, showing that the conclusion is reasonable.

$$\overline{b_2} = -\frac{\ln \varepsilon}{\overline{v_2}} = \frac{n}{\sum_{j=1}^{n} \frac{1}{M_j m_j}} \tag{1}$$

$$\overline{b_3} = -\frac{\ln \varepsilon}{\overline{v_3}} = \frac{\sum_{j=1}^{n} M_j m_j}{n} \quad (2)$$

We agree with the reviewer that an improvement on Eq. 1 would give better results. But in this case, the direct use of the new method for average visibility is probably a better choice.

3. Questions 3: I do not agree with the author's statement - "The argument that "there is no evidence that previous studies miscalculated visibility" does not lead to the conclusion that the algorithm for calculating the average visibility in the past is correct, nor to the conclusion that the title of the manuscript is misleading." Without any scientific evidence how can the author claim that all the previous methods were wrong? I liked the philosophical statement but it doesn't add anything to support their conclusion.

Response: The point we were trying to make at the time was that there are no conclusions disproving our point of views at the philosophical level, so the discussion should focus on the specific physical process and corresponding argumentative process.

4. Comment 1: Introduction section lacks background and references that should be addressed.

Response: Suggested changes have been implemented in the manuscript, where the concluding statement has been removed, underlying ideas for proposing the method is provided, and a description of the differences between the two methods has been added.

5. Comment 2: Inferences – I don't understand how the differences in equation 7 from equations 6 prove that the algorithm of equation 2 is wrong. It's just a case of two different ways of calculation. Here authors need solid justification.

Response: Thanks for pointing out this. It is true that the differences in equation 7 from equation 6 do not prove that the algorithm of equation 2 is wrong. There is a mistake in the sentence, and it should have been the comparison of Eq. 8 and Eq. 6 in the original manuscript, i.e., the Eq. 1 and Eq. 2 in the response to the Question 1 in this response to the reviewer. The expression in Eq. 1 contradicts the definition of the extinction coefficient, while the expression in Eq. 2 is in line with the definition of the extinction coefficient; therefore, Eq.2 is the reasonable calculation method. The corresponding text has been revised, please refer to the revised manuscript (P. 9, Line 185-195).

6. Comment 3: I agree that visibility mainly depends on particles, although studies have shown that gases have non-negligible (about 4-10%) contribution to visibility that needs to be considered.

Response: Thanks for pointing out this. Considering that the gas component also has a mass and a mass extinction coefficient, there is no need to discuss it separately and the gas component has been taken into account in the revised manuscript.

7. Comment 4: Relative error caused by the erroneous algorithm – This section needs more justifications. Simply, visibility can be calculated using the extinction coefficient at the individual point or the other way around. Furthermore, rather than being called an error, it shows more of a difference in values from methods 2 and 3. Here authors should provide clear and detailed descriptions of equation 9 and results. More experimental (mathematical) examples can help in this direction.

Response: Thanks for pointing out this. We now use the word "relative deviation" rather than "relative error" to describe the difference between the two methods. The section describing the relative deviation has been moved to an earlier position to illustrate the differences between the two methods.

8. Comment 5: Conclusions – This section needs some improvement. Authors are repeating the same message over and over. They should include possible limitations and improvements for both methods.

Response: Suggested changes have been implemented in the manuscript We have revised the conclusions, which now reads "This study proposes a new method for calculating the average of visibility data, i.e. harmonic average visibility. The main differences between the proposed harmonic average visibility from the previously used arithmetic average visibility are as follows.

1. The numerical values of harmonic average visibility and arithmetic average visibility are different. The values of harmonic average visibility are always smaller than the corresponding arithmetic average visibility, and the difference between them becomes larger as the observed visibility values fluctuate more strongly. Therefore, the method for calculating the average visibility should be carefully selected when analyzing large-scale or long-term visibility data, and when analyzing local visibility data with large changes in visibility within a short period of time.

2. Compared to the arithmetic average visibility, the harmonic average visibility can better represent changes in average atmospheric extinction coefficients and average aerosol concentrations. Therefore, we recommend preferentially using harmonic average visibility when calculating the average of visibility data in research related to climate change, atmospheric radiation, atmospheric pollution, environmental health, etc.".

General Comments: Overall, I am still not convinced by the authors' claim that the previous methods were wrong and that their method is the only correct method. I still believe that this paper only provides an additional method to calculate visibility, nothing more.

General Response: Many thanks again for the comments and suggestions, which help us to restrict our focus to the scientific issues, and avoid pointless arguments about the correctness of the previously used and the proposed method.

Response to reviewers for manuscript AMT-2022-74: **Average visibility that has been miscalculated**

We appreciate the editorial team and the reviewers for their time and comments towards improving our manuscript. We have revised the manuscript carefully, and we respond to all of the reviewers' points below. Responses are given in red.

Anonymous Referee #2

General Comments:

Zhang et al. report a reassessment of the equations used to calculate the average visibility and propose that the usual understanding of average visibility be dropped in favour of a weighted average. Their work highlights the important connection between the visibility and the underlying atmospheric extinction coefficient, which itself is connected to the composition of the atmosphere (particle concentration, composition, hygroscopicity, etc). Users of visibility data need to be alert to the differences between Eqs. (2) and (3), and that average visibility cannot be straightforwardly related to average extinction coefficient.

The authors do well to bring this to the attention of the community, but their approach has several flaws that should be addressed. I do not think that the work is publishable in its present form. It is not simply a question of the title, an issue raised by the two reviewers. The authors need to revise and clarify the argument and points made in the manuscript.

General response: Many thanks to the reviewer for the helpful comments. This time we have made substantial changes to the manuscript to address the reviewers' comments. Specifically, we restrict our focus to proposing a new method for calculating average visibility, rather than dwelling on proving that the old method is incorrect and the new method is correct. We have revised the title, the structure, the argumentative process, and the conclusion of the manuscript. Due to the number of changes, we did not highlight the revised sentences with a different colour. Detailed responses to the comments are below.

1. Comment 1: A major objection is that "average visibility" (Eq. 2) conforms to the accepted definition of arithmetic average or arithmetic mean of a given property, and this is what most scientists would understand by the term. The authors are right that Eq. (2) and (3) are not mathematically equivalent. What is needed then is for the authors to provide helpful terminology distinguishing between Eq. (2) ("average visibility") and Eq. (3). There are presumably circumstances when researchers would find the average visibility (Eq. 2) a useful concept for describing their observations. In other cases, a weighted average or some other statistic would be more appropriate (using Eq. (3) to study the underlying extinction coefficient). The circumstances for using one or other statistic needs to be clarified.

Because it may be reasonable to use one or other equation depending on a study's goals, the manuscript's terminology denoting logical conclusions ("therefore", "proves"), and correctness ("correct", "miscalculated", "error") is too narrow. This paper would be much more valuable to the scientific community if it brought greater clarity and nuance to the ways in which visibility data are analysed.

Response : The key question here is whether calculating the average of visibility data is a mathematical or physical problem. We have always considered it a physical problem. And we think a physical problem should have its physical meaning and corresponding process.

The previously used method carries out the summation over the visibility when calculating the average of visibility data. The visibility is neither an extensive quantity nor an intensity quantity, so that the results of the summation of visibility are just numerical values with no corresponding physical process. Therefore, the arithmetic average visibility calculated by the previously used method has no real physical meaning, leading to problems in the average extinction coefficient obtained using arithmetic average visibility. The revised manuscript elaborates on this point. On this basis, the new calculation method is justified from the point of view of physical processes, and the argument continues to explain why the harmonic average visibility calculated by the new method better reflects the changes in average atmospheric extinction coefficients and aerosol concentrations.

We agree that if the purpose is to numerically describe the measured visibility data, then the calculation of average visibility can be treated as a mathematical problem; therefore, it cannot be simply assumed that the previously used method is "incorrect". So the expressions "correct" or "incorrect" are not used in the revised manuscript. We turn to emphasize that the new method is more appropriate for calculating the average visibility when the average visibility is used as a parameter to characterize changes in atmospheric extinction coefficients and aerosol concentrations.

2. Comment 2: More emphasis should be placed on when the values produced by the two equations differ. The values produced by Eq. (2) and (3) converge as the range of visibility values becomes increasingly narrow. As Fig. 1c shows, the variation in the results of the two approaches within the hourly dataset is small, presumably because visibility is generally changing little over this time period for most observations. Greater variation occurs when datasets are long enough to contain larger variations in the individual visibility measurements.

Response: Thanks for pointing out this. The section describing the relative deviation has been moved to an earlier position to illustrate the differences between the two methods. We strengthen our analysis on the course of the difference, and emphasize that it is important to select the appropriate method for calculating the average visibility when analyzing large-scale or long-term visibility data, and when analyzing local visibility data with large changes in visibility within a short period of time.

3. Comment 3: The manuscript's arguments about the average value becoming infinite if any one of the measurement series is infinite are not convincing. Such contributions to the average visibility (or another physical property) do not occur in practice because individual measurements giving infinity are physically implausible (whether for visibility or another physical property) and would be removed from the data set. The formal mathematical possibility of an infinite result is not helpful.

Response : We agree that data at infinity can be used as a clue to a problem, but not as a basis for proof. We have removed the analysis of data at infinity and strengthened the theoretical analysis of the physical processes, giving a more general view of the conclusions.

4. Comment 4: The manuscript title is uninformative and potentially misleading. Both reviewers thoroughly disliked the title, but the authors were resistant to changing it. I think they should. Perhaps something like “Average visibility and its relationship to atmospheric extinction: a clarification” provides a better summary of their paper’s content and aims.

Response: Thanks for this comment. Suggested changes have been implemented in the manuscript, where the title has been changed to “A new method for calculating average visibility: from the relationship between extinction coefficient and visibility”.

A new method for calculating average visibility: from the relationship between extinction coefficient and visibility

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Abstract. Visibility data are fundamental meteorological observation data widely used in many fields. When using visibility data, it is often necessary to calculate the average visibility, which used to be the arithmetic average of the visibility data directly. In this study, we first analyze the relationship between the visibility, the extinction coefficient, and atmospheric compositions. Then we propose to use the harmonic average of visibility data as the average visibility, which can better reflect changes in atmospheric extinction coefficients and aerosol concentrations. It is recommended to use the harmonic average visibility in the studies of climate change, atmospheric radiation, air pollution, environmental health, etc.

1 Introduction

Visibility is a fundamental meteorological parameter (WMO, 1957, 2018) and has a wide range of application scenarios. On the on hand, as an indicator of atmospheric transparency, visibility data are used in many aspects of daily life, such as ground transportation (Ashley et al., 2015; Peng et al., 2017), aviation (Herzegh et al., 2015), and navigation (Debortoli et al., 2019), and in scientific research related to weather processes, such as the study of the formation and dissipation of fog. On the other hand, because visibility (v) is determined as a function of the atmospheric extinction coefficient (b) at a given contrast threshold (ε) (Koschmieder, 1924) (Eq. 1), and because the extinction coefficient is predominantly determined by aerosol concentrations (Che et al., 2007), visibility can also be used as a parameter describing atmospheric extinction coefficients (Zhang et al., 2017; Field et al., 2009) and aerosol concentrations (Rosenfeld et al., 2007; Chen et al., 2005), which is widely used in research related to climate change (Rosenfeld et al., 2007; Vautard et al., 2009), atmospheric radiation (Wang et al., 2009;

Wu et al., 2014), atmospheric pollution (Gunthe et al., 2021; Yang et al., 2017) and environmental health (Huang et al., 2009; Laden et al., 2006).

$$v = -\frac{\ln \varepsilon}{b} \tag{1}$$

A large amount of gridded visibility data have been accumulated through long-term observations at dense measurement sites (Pitchford et al., 2007; Singh et al., 2017), which greatly support many research. Calculating the average visibility is the most frequently performed task when using visibility data (An et al., 2019; Kessner et al., 2013; Zhang et al., 2010). It is easy to see how problems in calculating the average visibility could affect the credibility of the conclusions reached in previous studies using visibility data. Therefore, it is necessary to discuss the method of calculating the average visibility.

There are two variables in Eq. 1, visibility and the extinction coefficient, from which two methods for calculating the average visibility can be derived. The first method directly calculates the arithmetic average of visibility data using Eq. 2, where $\overline{v_2}$ represents the arithmetic average of visibility data, n is the number of measurements, and v_i denotes the visibility obtained in the i^{th} measurement. As can be seen from Eq. 2, the average visibility calculated by the first method is the arithmetic average visibility.

$$\overline{v}_2 = \frac{\sum_{i=1}^{n} v_i}{n} \tag{2}$$

The second method calculates the average extinction coefficient data first, then substitutes the average extinction coefficient into Eq. 1 to obtain the average visibility; the specific derivation process and results are shown in Eq. 3. Specifically, first, substitute the visibility measurement v_i into Eq. 1 to obtain the corresponding extinction coefficient b_i in the i^{th} measurement. Then, calculate the arithmetic average of a total of n extinction coefficients, denoted as $\bar{b}$. Finally, substitute the average extinction coefficient into Eq. 1 to obtain the average visibility $\overline{v_3}$. As can be seen from Eq. 3, the average visibility calculated by the second method is the harmonic average visibility.

$$b_i = \frac{\ln \varepsilon}{v_i} \Rightarrow \bar{b} = \frac{\sum_{i=1}^{n} b_i}{n} = -\frac{\sum_{i=1}^{n} \frac{\ln \varepsilon}{v_i}}{n} = -\frac{\ln \varepsilon}{n} \sum_{i=1}^{n} \frac{1}{v_i} \Rightarrow \overline{v}_3 = -\frac{\ln \varepsilon}{\bar{b}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{v_i}} \tag{3}$$

Equation 2 gives the arithmetic average visibility and Eq. 3 gives the harmonic average visibility. It is clear that the values of average visibility calculated by the two methods are different. This is because atmospheric visibility is constantly changing, and it has been mathematically proven that, unless all values used to calculate the average are the same, the arithmetic average is always greater than the harmonic average (Ferger, 1931).

The question arises as to whether the average visibility used in practical work should be the arithmetic average visibility calculated by Eq. 2 or the harmonic average visibility calculated by Eq. 3. To date, arithmetic average visibility has been used in studies (An et al., 2019; Kessner et al., 2013; Rosenfeld et al., 2007; Singh et al., 2017; Zhang et al., 2017) and harmonic average visibility has never been an option, so that when studies refer to average visibility, it is calculated directly using Eq. 2 without the need for clarification. The answer seems clear, but not yet convincing. This is because no theoretical justification has been given in past studies for using the arithmetic average visibility rather than the harmonic average visibility. Although it is true that the arithmetic average visibility is more intuitive, this does not exclude the possibility that the option of the harmonic average visibility has been overlooked in the past due to the blind spot in thinking. Therefore, a more in-depth discussion is necessary.

The first thing to do is to compare the difference in numerical values of the average visibility obtained by the two methods. If the difference is negligible, there is no point in discussing this issue, and the arithmetic average visibility obtained from Eq. 2 is also reliable. However, if the difference is considerable, it is necessary to analyze the difference in physical meaning between arithmetic average visibility and harmonic average visibility, and then select the appropriate calculation method for average visibility in different scenarios in combination with the purpose of using visibility data.

2 The numerical difference between arithmetic average visibility and harmonic average visibility

To develop an intuitive understanding of the magnitude of the numerical difference between arithmetic average visibility and harmonic average visibility, we analyzed the visibility data measured at 1-min resolution by a CJY-1 visibility meter (CAMA Measurement & Control Equipments Co., Ltd) on the campus of the Nanjing University of Information Science and Technology in Nanjing, China, during 2010–2017. The details regarding the observation site and instruments are given in Zhang et al. (2017).

The hourly, daily, monthly, and yearly arithmetic average visibility and harmonic average visibility are shown in Fig. 1a and 1b, respectively. By substituting the values of average visibility during the corresponding period shown in Fig. 1a and Fig. 1b into Eq. 4, we obtain the relative deviation of the hourly, daily, monthly, and yearly arithmetic average visibility from harmonic average visibility. Figure 1c shows the distribution of the magnitude of relative deviation. The value of 96.3 in the lower-left corner of Fig. 1c indicates that 96.3% of the relative deviation of the hourly average visibility falls within the range of 0–10%.

$$X\% = \frac{\overline{v_2} - \overline{v_3}}{\overline{v_3}} \times 100\% \tag{4}$$

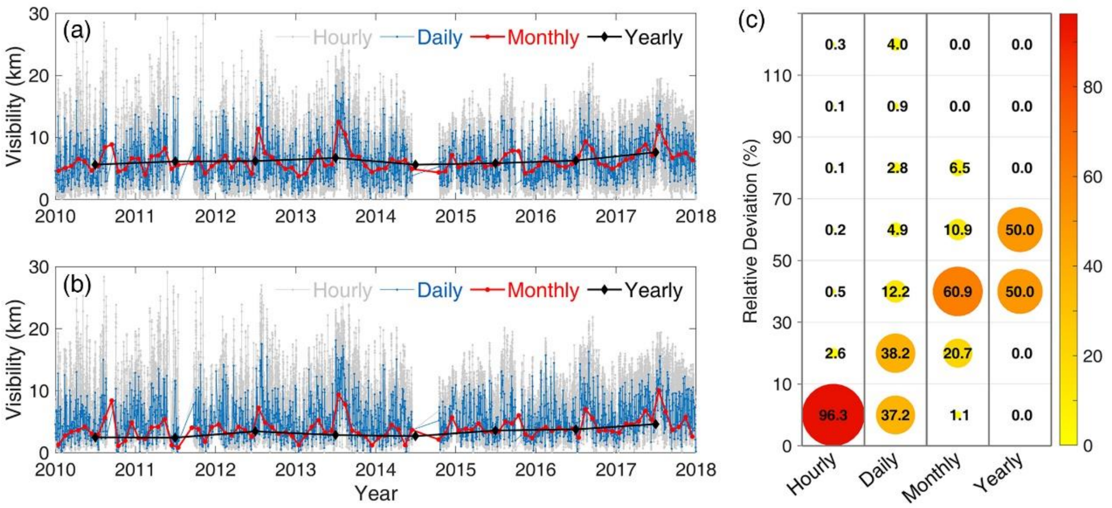

Figure 1: Comparison of arithmetic average visibility and harmonic average visibility: (a) arithmetic average visibility calculated using Eq. 2. (b) harmonic average visibility calculated using Eq. 3. (c) distribution of the relative deviation of arithmetic average visibility from harmonic average visibility.

As shown in Fig. 1, the arithmetic average visibility calculated using Eq. 2 (Fig. 1a) is always higher than the harmonic average visibility calculated using Eq. 3 (Fig. 1b); therefore, all values of the relative deviation lie in the range of greater than zero. The results in Fig. 1 are not a coincidence because of the specificity of the measurement data, but an inevitable result that will appear when calculating the average of any visibility measurement data using Eq. 2 and Eq. 3. It has been mathematically proven that, unless all values used to calculate the average are the same, the arithmetic average is always greater than the harmonic average; the greater the variation in the data, the greater the difference between the two.

The relationship between the arithmetic average and the harmonic average can explain the distribution of relative deviation values in Fig. 1c. The range of the measured visibility values is typically related to the observation period. The longer the duration of the observation, the larger the range of the measured visibility data. Therefore, the longer the observation period chose to calculate the average visibility, the larger the relative deviation of the arithmetic average visibility from the harmonic average visibility. It is not difficult to understand why the relative deviation of the yearly average is larger than that of the monthly average, which is larger than that of the hourly average, according to the distribution of the relative deviation shown in Fig. 1c.

Regarding the relative deviation of yearly and monthly arithmetic average visibility from harmonic average visibility (Fig. 1c), most of the values fall within the range of 30% to 70%, which is far greater than the typical range of measurement error of visibility meters (WMO, 2018). Regarding the relative deviation of hourly and daily average visibility, although most of the values are less than 30%, this does not mean that the difference between the arithmetic average and the harmonic average can be ignored. Because atmospheric visibility can sometimes change significantly in a short time, a topic of particular interest in previous studies, at which time the average visibility calculated by the two methods can be quite different.

In summary, as long as the atmospheric visibility is variable, the values of arithmetic average visibility and harmonic average visibility will not be the same, and the magnitude of the difference between them is related to the intensity of the change in visibility. Therefore, the difference between the two calculation methods cannot be ignored in large-scale and long-term studies. Even for small-scale and short-term studies, the difference is not negligible when there is a significant change in visibility.

3 Discussion of the physical meaning of the two calculation methods of average visibility

3.1 Discussion of the extinction coefficient and visibility

To understand the difference in physical meaning between arithmetic average visibility and harmonic average visibility, it is necessary to understand the characteristics of the two physical quantities, extinction coefficient and visibility. To this end, we design a thought experiment.

Assume there is a system, where there are a total of *n* substances affecting the extinction coefficient. The mass concentration of the i^{th} substance is m_i and the mass extinction coefficient is M_i. We carry out a thought experiment, and the experimental procedures and corresponding results are recorded in Table 1.

Table 1. Records of the thought experiment process

Experimental procedure	Extinction coefficient	Visibility
1. Remove all substances from the system	0	$+\infty$
2. Add the first substance to the system	$m_1 M_1$	$-\frac{\ln \varepsilon}{m_1 M_1}$
3. Continue adding the second substance to the system	$m_1 M_1 + m_2 M_2$	$-\frac{\ln \varepsilon}{m_1 M_1 + m_2 M_2}$
4. Continue adding the i^{th} substance to the system	$m_1 M_1 + m_2 M_2 + \cdots + m_i M_i$	$-\frac{\ln \varepsilon}{m_1 M_1 + m_2 M_2 + \cdots + m_i M_i}$
5. Repeat the above until all *n* substances are added to the system	$\sum_{i=1}^{n} m_i M_i$	$-\frac{\ln \varepsilon}{\sum_{i=1}^{n} m_i M_i}$

Two conclusions can be drawn from the results of the thought experiment recorded in Table 1. It should be noted that these two conclusions are not new knowledge but the basis for subsequent discussion.

The first conclusion is that the concentration and the optical properties of the substances determine the extinction coefficient and the visibility of the system. This suggests that the changes in the extinction coefficient and visibility of the system should logically match the changes in the mass concentration and mass extinction coefficient of the substances in the system.

The second conclusion is that the extinction coefficient is an extensive quantity, whereas the visibility is neither an extensive nor an intensity quantity. This is because the extinction coefficient is proportional to the amount of matter in the system, suggesting that the extinction coefficient is an extensible quantity. The visibility decreases as the amount of matter in the system increases, suggesting that visibility is not an extensible quantity. The magnitude of visibility varies with the concentration of the substance in the system, so it is not a characteristic property of the substance and not an intensity quantity. Therefore, the summation of visibility has no real physical meaning.

3.2 Discussion on the physical meaning of arithmetic average visibility and harmonic average visibility

Simulated measurements are generated in order to discuss the physical meaning of arithmetic average visibility and harmonic average visibility. Assuming that a total of *n* measurements are made with the same instrument, at the same site, at the same time interval, and the measurement results are considered reliable, Eq.5 relates the mass concentration (m_j) and the mass extinction coefficient (M_j) of substances to the extinction coefficient, and to the visibility in the j^{th} observation.

$$M_j m_j = b_j = -\frac{\ln \varepsilon}{v_j} \tag{5}$$

Then we calculate the average extinction coefficient and average visibility with three methods, respectively.

Method 1. Based on the first conclusion in section 3.1, the average extinction coefficient and average visibility were calculated using the concentrations and optical properties of the substances during the observation period, as the definition implies. First, calculate the average mass concentration and the average mass extinction coefficient during the observation period, as shown in Eq. 6. Then, calculate the average extinction coefficient and average visibility using the average mass concentration and the average mass extinction coefficient during the observation period, as shown in Eq. 7.

$$\overline{m} = \frac{\sum_{j=1}^{n} m_j}{n}, \ \overline{M} = \frac{\sum_{j=1}^{n} M_j m_j}{\sum_{j=1}^{n} m_j} \tag{6}$$

$$\bar{b} = \overline{m}\overline{M} = \frac{\sum_{j=1}^{n} M_j m_j}{n}, \ \bar{v} = -\frac{\ln \varepsilon}{\bar{b}} = -\frac{\ln \varepsilon}{\overline{m}\overline{M}} = -\frac{n \ln \varepsilon}{\sum_{j=1}^{n} M_j m_j} \tag{7}$$

Method 2. Substitute the observed mass concentration and mass extinction coefficient into Eq. 2 to obtain the arithmetic average visibility, which is then substituted into Eq. 1 to obtain the corresponding average extinction coefficient, as shown in Eq. 8.

$$\overline{v_2} = \frac{\sum_{j=1}^{n} v_j}{n} = \frac{\sum_{j=1}^{n} \frac{-\ln \varepsilon}{b_j}}{n} = -\frac{\ln \varepsilon}{n} \sum_{j=1}^{n} \frac{1}{M_j m_j}, \ \overline{b_2} = -\frac{\ln \varepsilon}{\overline{v_2}} = \frac{n}{\sum_{j=1}^{n} \frac{1}{M_j m_j}} \tag{8}$$

Method 3. Substitute the observed mass concentration and mass extinction coefficient into Eq. 3 to obtain the harmonic average visibility, which is then substituted into Eq. 1 to obtain the corresponding average extinction coefficient, as shown in Eq. 9.

$$\overline{v_3} = \frac{n}{\sum_{j=1}^{n} \frac{1}{v_j}} = \frac{n}{\sum_{j=1}^{n} \frac{b_j}{-\ln \varepsilon}} = -\frac{n \ln \varepsilon}{\sum_{j=1}^{n} b_j} = -\frac{n \ln \varepsilon}{\sum_{j=1}^{n} M_j m_j}, \ \overline{b_3} = -\frac{\ln \varepsilon}{\overline{v_3}} = \frac{\sum_{j=1}^{n} M_j m_j}{n} \tag{9}$$

The average visibility and average extinction coefficient calculated by the three methods are now compared and analyzed. A comparison of Eq. 7 and Eq. 9 indicates that the expression of the average visibility and the expression of the average extinction coefficient are identical respectively, while the expressions in Eq.8 are different from those in Eq. 7 and Eq. 9.

The reason for this can be explained by the second conclusion given in Section 3.1. All three methods perform summation. The Method 1 and Method 3 both carry out the summation over extensive quantities, i.e. the mass and the extinction coefficient, so that their corresponding physical meanings are clear. The Method 1 and Method 3 actually describe the same physical process, i.e. the mixing process. However, the Method 2 carries out the summation over the visibility, which is neither an extensive quantity nor an intensity quantity, so that the results of the summation of visibility are just numerical values with no corresponding physical process. Therefore, the arithmetic average visibility has no real physical meaning.

The difference in the physical meaning of arithmetic average visibility and harmonic average visibility leads to the difference in the derived expressions of the average extinction coefficient. It can be seen from Eq.7 and Eq.9 that the expression of the average extinction coefficient derived from the harmonic average visibility (Eq.9) is identical to that derived from the definition of the extinction coefficient (Eq.7). However, a comparison of Eq. 7 and Eq. 8 indicates that the expression of the average extinction coefficient derived from the arithmetic average visibility (Eq.8) differs from that derived from the definition of the extinction coefficient (Eq.7). This suggests that we should use the harmonic average visibility rather than the arithmetic average visibility when using average visibility data to obtain average extinction coefficient. Considering that the main contribution to atmospheric extinction comes from aerosol particles, it is also appropriate to use harmonic average visibility data for research on aerosols using visibility data.

In summary, if the purpose is to numerically describe the measured visibility data, then the calculation of average visibility can be treated as a mathematical problem, and the arithmetic average visibility can be used to represent the average visibility. However, if the average visibility is used as a parameter to characterize changes in atmospheric extinction coefficients and aerosol concentrations, especially in research related to climate change, atmospheric radiation, atmospheric pollution, environmental health, etc., then the calculation of average visibility should be treated as a physical problem, and the harmonic average visibility should be used to represent the average visibility.

4 Conclusions

This study proposes a new method for calculating the average of visibility data, i.e. harmonic average visibility. The main differences between the proposed harmonic average visibility from the previously used arithmetic average visibility are as follows.

1. The numerical values of harmonic average visibility and arithmetic average visibility are different. The values of harmonic average visibility are always smaller than the corresponding arithmetic average visibility, and the difference between them becomes larger as the observed visibility values fluctuate more strongly. Therefore, the method for calculating the average visibility should be

carefully selected when analyzing large-scale or long-term visibility data, and when analyzing local visibility data with large changes in visibility within a short period of time.

2. Compared to the arithmetic average visibility, the harmonic average visibility can better represent changes in average atmospheric extinction coefficients and average aerosol concentrations. Therefore, we recommend preferentially using harmonic average visibility when calculating the average of visibility data in research related to climate change, atmospheric radiation, atmospheric pollution, environmental health, etc.

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Data availability

The data supporting the conclusions have been deposited in Zenodo (https://doi.org/10.5281/zenodo.5025882).

References

An, Z., Huang, R.-J., Zhang, R., Tie, X., Li, G., Cao, J., Zhou, W., Shi, Z., Han, Y., Gu, Z., and Ji, Y.: Severe haze in northern China: A synergy of anthropogenic emissions and atmospheric processes, Proc. Natl. Acad. Sci. U. S. A., 116, 8657–8666, doi:10.1073/pnas.1900125116, 2019.

Ashley, W. S., Strader, S., Dziubla, D. C., and Haberlie, A.: Driving blind: Weather-related vision hazards and fatal motor vehicle crashes, B. Am. Meteorol. Soc., 96, 755–778, doi:10.1175/bams-d-14-00026.1, 2015.

Che, H., Zhang, X., Li, Y., Zhou, Z., and Qu, J. J.: Horizontal visibility trends in China 1981–2005, Geophys. Res. Lett., 34, L24706, doi:10.1029/2007gl031450, 2007.

Chen, L. H., Knutsen, S. F., Shavlik, D., Beeson, W. L., Petersen, F., Ghamsary, M., and Abbey, D.: The Association between Fatal Coronary Heart Disease and Ambient Particulate Air Pollution: Are Females at Greater Risk?, Environ. Health Perspect., 113, 1723-1729, doi: 10.1289/ehp.8190, 2005.

Debortoli, N. S., Clark, D. G., Ford, J. D., Sayles, J. S., and Diaconescu, E. P.: An integrative climate change vulnerability index for Arctic aviation and marine transportation, Nat. Commun., 10, 2596, doi:10.1038/s41467-019-10347-1, 2019.

Field, R. D., van der Werf, G. R., and Shen, S. S. P.: Human amplification of drought-induced biomass burning in Indonesia since 1960, Nat. Geosci., 2, 185–188, doi:10.1038/ngeo443, 2009.

Gunthe, S. S., Liu, P., Panda, U., Raj, S. S., Sharma, A., Darbyshire, E., Reyes-Villegas, E., Allan, J., Chen, Y., Wang, X., Song, S., Pöhlker, M. L., Shi, L., Wang, Y., Kommula, S. M., Liu, T., Ravikrishna, R., McFiggans, G., Mickley, L. J., Martin, S. T., Pöschl, U., Andreae, M. O., and Coe, H.: Enhanced aerosol particle growth sustained by high continental chlorine emission in India, Nat. Geosci., 14, 77–84, doi:10.1038/s41561-020-00677-x, 2021.

Ferger, W. F.: The Nature and Use of the Harmonic Mean, J. Am. Stat. Assoc., 26, 36-40, doi: 10.1080/01621459.1931.10503148, 1931.

Herzegh, P., Wiener, G., Bateman, R., Cowie, J., and Black, J.: Data fusion enables better recognition of ceiling and visibility hazards in aviation, B. Am. Meteorol. Soc., 96, 526–532, doi:10.1175/bams-d-13-00111.1, 2015.

Huang, W., Tan, J., Kan, H., Zhao, N., Song, W., Song, G., Chen, G., Jiang, L., Jiang, C., Chen, R., and Chen, B.: Visibility, air quality and daily mortality in Shanghai, China, Sci. Total Environ., 407, 3295–3300, doi:https://doi.org/10.1016/j.scitotenv.2009.02.019, 2009.

Kessner, A. L., Wang, J., Levy, R. C., and Colarco, P. R.: Remote sensing of surface visibility from space: A look at the United States East Coast, Atmos. Environ., 81, 136–147, doi:https://doi.org/10.1016/j.atmosenv.2013.08.050, 2013.

Koschmieder, H.: Theorie der horizontalen sichtweite, Beiträge zur Physik der freien Atmosphäre, Meteorol. Z., 12, 33–55, 1924.

Laden, F., Schwartz, J., Speizer, F. E., and Dockery, D. W.: Reduction in fine particulate air pollution and mortality, Am. J. Respir. Crit. Care Med., 173, 667–672, doi:10.1164/rccm.200503-443OC, 2006.

Peng, Y., Abdel-Aty, M., Shi, Q., and Yu, R.: Assessing the impact of reduced visibility on traffic crash risk using microscopic data and surrogate safety measures, Transp. Res. C-Emer., 74, 295–305, doi:https://doi.org/10.1016/j.trc.2016.11.022, 2017.

Pitchford, M., Malm, W., Schichtel, B., Kumar, N., Lowenthal, D., and Hand, J.: Revised algorithm for estimating light extinction from IMPROVE particle speciation data, J. Air Waste Manag. Assoc., 57, 1326–1336, doi:10.3155/1047-3289.57.11.1326, 2007.

Rosenfeld, D., Dai, J., Yu, X., Yao, Z., Xu, X., Yang, X., and Du, C.: Inverse relations between amounts of air pollution and orographic precipitation, Science, 315, 1396–1398, doi:10.1126/science.1137949, 2007.

Singh, A., Bloss, W. J., and Pope, F. D.: 60 years of UK visibility measurements: Impact of meteorology and atmospheric pollutants on visibility, Atmos. Chem. Phys., 17, 2085–2101, doi:10.5194/acp-17-2085-2017, 2017.

Vautard, R., Yiou, P., and van Oldenborgh, G. J.: Decline of fog, mist and haze in Europe over the past 30 years, Nat. Geosci., 2, 115–119, doi:10.1038/ngeo414, 2009.

Wang, K., Dickinson, R. E., and Liang, S.: Clear sky visibility has decreased over land globally from 1973 to 2007, Science, 323, 1468–1470, 2009.

WMO: Ninth Session of the Executive Committee: Abridged Report with Resolutions (WMO-No. 67), World Meteorological Organization, Geneva, Switzerland, 1957.

WMO: Guide to instruments and methods of observation, Volume I – Measurement of Meteorological Variables, World Meteorological Organization, Geneva, Switzerland, 2018.

Wu, J., Luo, J., Zhang, L., Xia, L., Zhao, D., and Tang, J.: Improvement of aerosol optical depth retrieval using visibility data in China during the past 50 years, J. Geophys. Res.-Atmos., 119, 13,370–13,387, doi:10.1002/2014jd021550, 2014.

Yang, Y., Russell, L. M., Lou, S., Liao, H., Guo, J., Liu, Y., Singh, B., and Ghan, S. J.: Dust-wind interactions can intensify aerosol pollution over eastern China, Nat. Commun., 8, 15333, doi:10.1038/ncomms15333, 2017.

Zhang, Q. H., Zhang, J. P., and Xue, H. W.: The challenge of improving visibility in Beijing, Atmos. Chem. Phys., 10, 7821–7827, doi:10.5194/acp-10-7821-2010, 2010.

Zhang, Z., Shen, Y., Li, Y., Zhu, B., and Yu, X.: Analysis of extinction properties as a function of relative humidity using a κ-EC-Mie model in Nanjing, Atmos. Chem. Phys., 17, 4147–4157, doi:10.5194/acp-17-4147-2017, 2017.