



## Average visibility that has been miscalculated

Zefeng Zhang, Hengnan Guo, Hanqing Kang, Jing Wang, Junlin An, Xingna Yu,  
Jingjing Lv, Bin Zhu

Key Laboratory for Aerosol-Cloud-Precipitation of China Meteorological Administration, Nanjing  
5 University of Information Science & Technology, Nanjing, 210044, China

*Correspondence to:* Zefeng Zhang (zhangzf01@vip.163.com)

**Abstract.** Visibility data are fundamental meteorological data widely used in many fields such as climate change, atmospheric radiation, atmospheric pollution, and environmental health. Calculating the average visibility is typically the first step when using visibility data. However, this study proves that the algorithm previously used to calculate average visibility is incorrect, leading to a non-negligible error in average visibility data. Moreover, the use of this incorrect algorithm not only artificially reduces the reliability of visibility data, but also affects the credibility and even the correctness of the conclusions reached in previous studies using visibility data. Therefore, we present the correct algorithm for average visibility, which should be applied to both future and previous research to significantly increase the reliability and application scope of visibility data.

### 1 Introduction

Visibility is a fundamental meteorological parameter (WMO, 1957, 2018). A large amount of visibility data has been accumulated through long-term observations at dense measurement sites (Pitchford et al., 2007; Singh et al., 2017). Changes in visibility not only influence aspects of daily life, such as ground transportation (Ashley et al., 2015; Peng et al., 2017), aviation (Herzogh et al., 2015), and navigation (Debortoli et al., 2019), but also have psychological effects on people's well-being (Li et al., 2018). As a parameter describing atmospheric extinction coefficients (Zhang et al., 2017; Field et al., 2009) and aerosol concentrations (Rosenfeld et al., 2007; Chen et al., 2005), visibility is widely used in research related to climate change (Rosenfeld et al., 2007; Vautard et al., 2009), atmospheric radiation (Wang et al., 2009; Wu et al., 2014), atmospheric pollution (Gunthe et al., 2021; Yang et al., 2017) and environmental health (Huang et al., 2009; Laden et al., 2006). This is because visibility ( $v$ ) is determined as a function of the atmospheric extinction coefficient ( $b$ ) at a given contrast threshold ( $\epsilon$ )



(Koschmieder, 1924) (Eq. 1), and the extinction coefficient is predominantly determined by the aerosol concentration (Che et al., 2007).

$$v = -\frac{\ln \varepsilon}{b} \quad (1)$$

Calculating the average visibility is the most frequently performed task when using visibility data (An et al., 2019; Kessner et al., 2013; Zhang et al., 2010). Two methods of calculating the average visibility arise from Eq. 1. The first method directly calculates the average of visibility data using the algorithm shown in Eq. 2. The second method calculates the average extinction coefficient data first, then substitutes the averaged extinction coefficient into Eq. 1 to obtain the average visibility; the corresponding algorithm is shown in Eq. 3.

$$\bar{v}_2 = \frac{\sum_{i=1}^n v_i}{n} \quad (2)$$

$$\bar{v}_3 = -\frac{\ln \varepsilon}{\bar{b}} \quad (3)$$

where  $\bar{v}_2$  and  $\bar{v}_3$  represent the average visibility calculated using Eq. 2 and Eq. 3, respectively,  $\bar{b}$  is the average extinction coefficient,  $n$  is the number of measurements, and  $v_i$  denotes the visibility obtained in the  $i^{\text{th}}$  measurement.

The question arises as to whether the average visibility values calculated by the algorithms of Eq. 2 and Eq. 3 are the same? If not, which is the correct algorithm? Unfortunately, the above questions have not previously been seriously discussed. Intuitively, Eq. 2 has been used as the correct algorithm to calculate the average visibility in previous studies (An et al., 2019; Kessner et al., 2013; Rosenfeld et al., 2007; Singh et al., 2017; Zhang et al., 2017), and Eq. 3 has never been discussed to calculate the average visibility. However, this study proves that Eq. 2 is incorrect, and should not be used to estimate other parameters, such as the concentration of PM<sub>2.5</sub> (Chen et al., 2005), aerosol optical depth (Wu et al., 2021), mortality (Huang et al., 2009), etc. Eq. 3 is instead the correct algorithm for calculating average visibility. Therefore, the reliability of both visibility observations and the results of previous



studies using visibility data has been artificially reduced by the continuous use of an incorrect algorithm to calculate the average visibility.

## 2 Inferences

To determine the correct algorithm between Eq. 2 and Eq. 3, it is necessary to discuss the physical meaning of both algorithms. Because atmospheric visibility is mainly determined by aerosol particles (Wang et al., 2009), to simplify the problem, only the effect of aerosol particles on visibility is considered in this study. Assuming that a total of  $n$  measurements are made at the same site with the same time interval, Eq. 4 relates the mass concentration ( $m$ ) and the mass extinction coefficient ( $M$ ) of aerosol particles to the extinction coefficient, and to the visibility in the  $i^{\text{th}}$  observation (Cheng et al., 2013).

$$M_i m_i = b_i = -\frac{\ln \varepsilon}{v_i} \quad (4)$$

It should be noted that it is the mass concentration and mass extinction coefficient of aerosol particles that determine the extinction coefficient and visibility of the atmosphere, not the other way around. Similarly, it is the average mass concentration and average mass extinction coefficient of aerosol particles during the observation period that determine the average extinction coefficient and average visibility during the observation period, not the other way around. Therefore, to calculate the average visibility during the observation period, we should first calculate the average mass concentration and the average mass extinction coefficient during the observation period, as shown in Eq. 5.

$$\bar{m} = \frac{\sum_{i=1}^n m_i}{n}, \bar{M} = \frac{\sum_{i=1}^n M_i m_i}{\sum_{i=1}^n m_i} \quad (5)$$

Then, we establish the relationship of the average mass concentration and average mass extinction coefficient to the average extinction coefficient and average visibility of the atmosphere. The result is shown in Eq. 6.



$$\overline{Mm} = \frac{\sum_{i=1}^n M_i m_i}{n} = \frac{\sum_{i=1}^n b_i}{n} = \bar{b} \Rightarrow \bar{v} = -\frac{\ln \varepsilon}{\bar{b}} = -\frac{\ln \varepsilon}{\overline{Mm}} \quad (6)$$

75 A comparison of Eq. 6 and Eq. 3 indicates that they are identical. Therefore, the algorithm of Eq. 3 is the correct algorithm for calculating the average visibility. The following is a discussion of whether the algorithm of Eq. 2 is the correct algorithm, which is characterized by direct calculation of the average visibility using observed visibility data. Equation 7 shows the relationship between the average visibility calculated from the algorithm of Eq. 2 and aerosol particles. Equation 8 gives the relationship  
 80 between the average extinction coefficient and aerosol particles.

$$\bar{v}_2 = \frac{\sum_{i=1}^n v_i}{n} = \frac{\sum_{i=1}^n \frac{-\ln \varepsilon}{b_i}}{n} = -\frac{\ln \varepsilon}{n} \sum_{i=1}^n \frac{1}{M_i m_i} \quad (7)$$

$$\bar{b} = -\frac{\ln \varepsilon}{\bar{v}_2} = \frac{n}{\sum_{i=1}^n \frac{1}{M_i m_i}} \quad (8)$$

The relationship of the average visibility and the average extinction coefficients to aerosol particles in Eq. 7 is significantly different from that in Eq. 6; therefore, the algorithm of Eq. 2 is  
 85 incorrect. The error in Eq. 2 occurs because visibility is treated as an independent variable rather than a function of aerosol particles. This affects the average value of visibility data by increasing the weight of visibility data at low aerosol concentrations and decreasing the weight of visibility data at high aerosol concentrations. As an extreme example, if the concentration of aerosol particles was zero in the  $i^{\text{th}}$  measurement, it follows from Eq. 7 and Eq. 8 that the average visibility obtained from the algorithm  
 90 of Eq. 2 would be infinitely large and the average extinction coefficient would be infinitely small, regardless of the concentration of aerosol particles in the other  $n-1$  measurements, which is clearly illogical.

This proves that Eq. 3 is the correct algorithm for calculating the average visibility, whereas Eq. 2 is incorrect. However, this does not necessarily indicate that previous average visibility values  
 95 calculated using Eq. 2 are not credible. Actual visibility observation data are required to compare the differences between the average visibility values calculated by Eq. 2 and Eq. 3. If the difference is



negligible, the average visibility obtained from Eq. 2 is also reliable. If the difference is considerable, then not only should the algorithm of Eq.2 not be used for future calculations of average visibility, but the corresponding results of previous studies should be revised.

### 100 3 Relative error caused by the erroneous algorithm

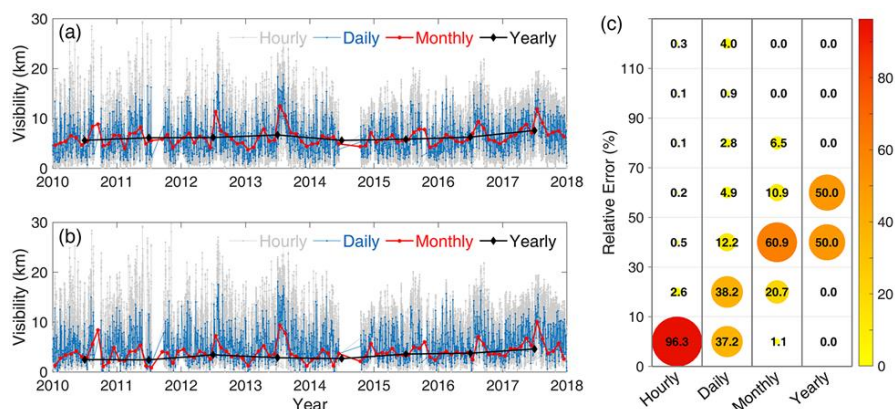
To develop an intuitive understanding of the magnitude of the relative error in average visibility values calculated using Eq. 2, we analyze the visibility data measured at 1-min resolution by a CJY-1 visibility meter (CAMA Measurement & Control Equipments Co., Ltd) on the campus of the Nanjing University of Information Science and Technology in Nanjing, China, during 2010–2017. The details  
 105 regarding the observation site and instruments are given in Zhang et al. (2017).

Typically, the output of a visibility meter is the value of visibility. Therefore, the average visibility is calculated directly from the output visibility by the algorithm of Eq. 2. However, more steps are required to derive the average visibility using the algorithm of Eq. 3. First, the extinction coefficient in the  $i^{\text{th}}$  measurement ( $b_i$ ) is derived by substituting the measured value of visibility ( $v_i$ ) into Eq. 1. Then,  
 110 the average extinction coefficient is calculated using a total of  $n$  extinction coefficients. The specific derivation process and results are shown in Eq. 9.

$$b_i = -\frac{\ln \varepsilon}{v_i} \Rightarrow \bar{b} = \frac{\sum_{i=1}^n b_i}{n} = -\frac{\sum_{i=1}^n \frac{\ln \varepsilon}{v_i}}{n} = -\frac{\ln \varepsilon}{n} \sum_{i=1}^n \frac{1}{v_i} \Rightarrow v_3 = -\frac{\ln \varepsilon}{\bar{b}} = \frac{n}{\sum_{i=1}^n \frac{1}{v_i}} \quad (9)$$

The hourly, daily, monthly, and yearly average visibility values calculated using Eq. 2 and Eq. 3 are shown in Fig. 1a and 1b, respectively. It is clear from the above discussion that Fig. 1a shows the  
 115 erroneous average visibility calculated by the incorrect algorithm, whereas Fig. 1b shows the average visibility calculated by the correct algorithm. By substituting the values of average visibility during the corresponding period shown in Fig. 1a and Fig. 1b into Eq. 10, we obtain the relative error of the hourly, daily, monthly, and yearly average visibility calculated by Eq. 2. Figure 1c shows the distribution of the magnitude of relative error. The value of 96.3 in the lower-left corner of Fig. 1c indicates that 96.3% of  
 120 the relative error of the hourly average visibility calculated by Eq. 2 falls within the range of 0–10%.

$$X\% = \frac{\bar{v}_2 - \bar{v}_3}{\bar{v}_3} \times 100\% \quad (10)$$



**Figure 1:** Comparison of average visibility calculated from the algorithms of Eq. 2 and Eq. 3: (a) average visibility calculated by the algorithm of Eq. 2. (b) average visibility calculated by the algorithm of Eq. 3. (c) distribution of the relative error of the average visibility calculated by Eq. 2.

As shown in Fig. 1, the average visibility calculated using Eq. 2 (Fig. 1a) is always higher than that calculated using Eq. 3 (Eq. 9) (Fig. 1b); therefore, all values of the relative error lie in the range of greater than zero. The results in Fig. 1 are not a coincidence because of the specificity of the measurement data, but an inevitable result that will appear when calculating the average of any visibility measurement data using the algorithms of Eq. 2 and Eq. 3. A more in-depth look at Eq. 2 and Eq. 9 (Eq. 3) reveals that Eq. 2 calculates the arithmetic mean of visibility, whereas Eq. 9 calculates the harmonic mean of visibility. It has been mathematically proven that, unless all values used to calculate the average are the same, the arithmetic mean is always greater than the harmonic mean; the greater the variation in the data, the greater the difference between the two (Ferber, 1931).

The relationship between the arithmetic mean and harmonic mean can explain the distribution of the relative error values in Fig. 1c. The range of the measured visibility values is typically related to the observation period. The longer the duration of observations, the larger the range of the measured visibility data. Therefore, the longer the observation period used to calculate the average visibility, the larger the relative error caused by the algorithm of Eq. 2. It is not difficult to understand why the relative error of the yearly average is larger than that of the monthly average, which is larger than that of the hourly average, according to the distribution of the relative error shown in Fig. 1c. Regarding the relative errors of yearly and monthly average visibility caused by the algorithm of Eq. 2 (Fig. 1c), most of the values



fall within the range of 30% to 70%, which is far greater than the typical range of measurement error of  
145 visibility meters (WMO, 2018). Therefore, the error caused by the incorrect algorithm of Eq. 2 cannot  
be ignored. Regarding the relative error of hourly and daily average visibility, although most of the values  
are less than 30%, this does not mean that the average visibility can be calculated by the algorithm of Eq.  
2 for short observation periods. Because sometimes the atmospheric visibility may change significantly  
in a short time, the relative error of the average visibility calculated by Eq. 2 is large over this time period.  
150 The largest relative errors in Fig. 1c caused by the algorithm of Eq. 2 fall into this category.

The only way to conclude that the average relative error caused by Eq. 2 is sufficiently small to  
continue using this algorithm, despite knowing that Eq. 2 is incorrect for calculating the average visibility,  
would be to perform statistical analysis of a large amount of visibility data obtained from different sites  
at different times. However, to reject this conclusion, it is logically enough to be able to provide a counter  
155 example. That is, the relative error range of the average visibility values calculated by Eq. 2 in this study  
(Fig. 1) is sufficient to show that the error in average visibility arising from the incorrect algorithm is not  
negligible.

#### 4 Conclusions

This study proves that the algorithm that has been used to calculate the average visibility is incorrect,  
160 and proves that the error in average visibility caused by the incorrect algorithm is not negligible. On this  
basis, the correct visibility algorithm is proposed in this study. The average visibility has so far been  
calculated from the incorrect algorithm, which will not only artificially reduce the reliability of visibility  
data, but also affect the credibility and even the correctness of the conclusions reached in the previous  
studies using visibility data. Therefore, not only should the correct algorithm be used to calculate the  
165 average visibility in the future, but also the past visibility data should be revised, as this will significantly  
increase the reliability of the visibility data and thus extend the range of applicability of the visibility  
data. In addition, the error in the algorithm for average visibility occurs because of inconsistencies  
between the measurement parameters and the target parameters. It cannot be excluded that similar  
problems occur in other instruments, so it is necessary to analyze the measurement principles of different  
170 instruments to avoid the recurrence of such errors.



### Acknowledgements

This work was supported by the National Key Research and Development Program of China (No. 2019YFC0214604). We thank WeiWei Wang, Li Xia and Jiade Yan for offering visibility data and maintaining the observation site used in this study.

### 175 Data availability

The data supporting the conclusions have been deposited in Zenodo (<https://doi.org/10.5281/zenodo.5025882>).

### References

- An, Z., Huang, R.-J., Zhang, R., Tie, X., Li, G., Cao, J., Zhou, W., Shi, Z., Han, Y., Gu, Z., and Ji, Y.: Severe haze in northern China: A synergy of anthropogenic emissions and atmospheric processes, *Proc. Natl. Acad. Sci. U. S. A.*, 116, 8657–8666, doi:10.1073/pnas.1900125116, 2019.
- Ashley, W. S., Strader, S., Dziubla, D. C., and Haberlie, A.: Driving blind: Weather-related vision hazards and fatal motor vehicle crashes, *B. Am. Meteorol. Soc.*, 96, 755–778, doi:10.1175/bams-d-14-00026.1, 2015.
- 185 Che, H., Zhang, X., Li, Y., Zhou, Z., and Qu, J. J.: Horizontal visibility trends in China 1981–2005, *Geophys. Res. Lett.*, 34, L24706, doi:10.1029/2007gl031450, 2007.
- Chen, L. H., Knutsen, S. F., Shavlik, D., Beeson, W. L., Petersen, F., Ghamsary, M., and Abbey, D.: The Association between Fatal Coronary Heart Disease and Ambient Particulate Air Pollution: Are Females at Greater Risk?, *Environ. Health Perspect.*, 113, 1723-1729, doi: 10.1289/ehp.8190, 2005.
- 190 Cheng, Z., Wang, S., Jiang, J., Fu, Q., Chen, C., Xu, B., Yu, J., Fu, X., and Hao, J.: Long-term trend of haze pollution and impact of particulate matter in the Yangtze River Delta, China, *Environ. Pollut.*, 182, 101–110, doi:https://doi.org/10.1016/j.envpol.2013.06.043, 2013.
- Debortoli, N. S., Clark, D. G., Ford, J. D., Sayles, J. S., and Diaconescu, E. P.: An integrative climate change vulnerability index for Arctic aviation and marine transportation, *Nat. Commun.*, 10, 2596, doi:10.1038/s41467-019-10347-1, 2019.
- Field, R. D., van der Werf, G. R., and Shen, S. S. P.: Human amplification of drought-induced biomass burning in Indonesia since 1960, *Nat. Geosci.*, 2, 185–188, doi:10.1038/ngeo443, 2009.





- Gunthe, S. S., Liu, P., Panda, U., Raj, S. S., Sharma, A., Darbyshire, E., Reyes-Villegas, E., Allan, J.,  
Chen, Y., Wang, X., Song, S., Pöhlker, M. L., Shi, L., Wang, Y., Kommula, S. M., Liu, T., Ravikrishna,  
200 R., McFiggans, G., Mickleby, L. J., Martin, S. T., Pöschl, U., Andreae, M. O., and Coe, H.: Enhanced  
aerosol particle growth sustained by high continental chlorine emission in India, *Nat. Geosci.*, 14, 77–84,  
doi:10.1038/s41561-020-00677-x, 2021.
- Herzogh, P., Wiener, G., Bateman, R., Cowie, J., and Black, J.: Data fusion enables better recognition of  
ceiling and visibility hazards in aviation, *B. Am. Meteorol. Soc.*, 96, 526–532, doi:10.1175/bams-d-13-  
205 00111.1, 2015.
- Huang, W., Tan, J., Kan, H., Zhao, N., Song, W., Song, G., Chen, G., Jiang, L., Jiang, C., Chen, R., and  
Chen, B.: Visibility, air quality and daily mortality in Shanghai, China, *Sci. Total Environ.*, 407, 3295–  
3300, doi:https://doi.org/10.1016/j.scitotenv.2009.02.019, 2009.
- Kessner, A. L., Wang, J., Levy, R. C., and Colarco, P. R.: Remote sensing of surface visibility from  
210 space: A look at the United States East Coast, *Atmos. Environ.*, 81, 136–147,  
doi:https://doi.org/10.1016/j.atmosenv.2013.08.050, 2013.
- Koschmieder, H.: Theorie der horizontalen sichtweite, *Beiträge zur Physik der freien Atmosphäre*,  
*Meteorol. Z.*, 12, 33–55, 1924.
- Laden, F., Schwartz, J., Speizer, F. E., and Dockery, D. W.: Reduction in fine particulate air pollution  
215 and mortality, *Am. J. Respir. Crit. Care Med.*, 173, 667–672, doi:10.1164/rccm.200503-443OC, 2006.
- Li, Y., Guan, D., Tao, S., Wang, X., and He, K.: A review of air pollution impact on subjective well-  
being: Survey versus visual psychophysics, *J. Clean. Prod.*, 184, 959–968,  
doi:https://doi.org/10.1016/j.jclepro.2018.02.296, 2018.
- Peng, Y., Abdel-Aty, M., Shi, Q., and Yu, R.: Assessing the impact of reduced visibility on traffic crash  
220 risk using microscopic data and surrogate safety measures, *Transp. Res. C-Emer.*, 74, 295–305,  
doi:https://doi.org/10.1016/j.trc.2016.11.022, 2017.
- Pitchford, M., Malm, W., Schichtel, B., Kumar, N., Lowenthal, D., and Hand, J.: Revised algorithm for  
estimating light extinction from IMPROVE particle speciation data, *J. Air Waste Manag. Assoc.*, 57,  
1326–1336, doi:10.3155/1047-3289.57.11.1326, 2007.
- 225 Rosenfeld, D., Dai, J., Yu, X., Yao, Z., Xu, X., Yang, X., and Du, C.: Inverse relations between amounts  
of air pollution and orographic precipitation, *Science*, 315, 1396–1398, doi:10.1126/science.1137949,  
2007.



- Singh, A., Bloss, W. J., and Pope, F. D.: 60 years of UK visibility measurements: Impact of meteorology and atmospheric pollutants on visibility, *Atmos. Chem. Phys.*, 17, 2085–2101, doi:10.5194/acp-17-2085-230 2017, 2017.
- Vautard, R., Yiou, P., and van Oldenborgh, G. J.: Decline of fog, mist and haze in Europe over the past 30 years, *Nat. Geosci.*, 2, 115–119, doi:10.1038/ngeo414, 2009.
- Wang, K., Dickinson, R. E., and Liang, S.: Clear sky visibility has decreased over land globally from 1973 to 2007, *Science*, 323, 1468–1470, 2009.
- 235 WMO: Ninth Session of the Executive Committee: Abridged Report with Resolutions (WMO-No. 67), World Meteorological Organization, Geneva, Switzerland, 1957.
- WMO: Guide to instruments and methods of observation, Volume I – Measurement of Meteorological Variables, World Meteorological Organization, Geneva, Switzerland, 2018.
- Wu, J., Luo, J., Zhang, L., Xia, L., Zhao, D., and Tang, J.: Improvement of aerosol optical depth retrieval 240 using visibility data in China during the past 50 years, *J. Geophys. Res.-Atmos.*, 119, 13,370–13,387, doi:10.1002/2014jd021550, 2014.
- Wu, J., Zhang, S., Yang, Q., Zhao, D., Fan, W., Zhao, J., and Shen, C.: Using particle swarm optimization to improve visibility-aerosol optical depth retrieval method, *npj Climate and Atmospheric Science*, 4, 49, doi:10.1038/s41612-021-00207-5, 2021.
- 245 Yang, Y., Russell, L. M., Lou, S., Liao, H., Guo, J., Liu, Y., Singh, B., and Ghan, S. J.: Dust-wind interactions can intensify aerosol pollution over eastern China, *Nat. Commun.*, 8, 15333, doi:10.1038/ncomms15333, 2017.
- Zhang, Q. H., Zhang, J. P., and Xue, H. W.: The challenge of improving visibility in Beijing, *Atmos. Chem. Phys.*, 10, 7821–7827, doi:10.5194/acp-10-7821-2010, 2010.
- 250 Zhang, Z., Shen, Y., Li, Y., Zhu, B., and Yu, X.: Analysis of extinction properties as a function of relative humidity using a  $\kappa$ -EC-Mie model in Nanjing, *Atmos. Chem. Phys.*, 17, 4147–4157, doi:10.5194/acp-17-4147-2017, 2017.