Response to the reviewers' comments

Reviewer 2

Abstract

**   Line 4: There is no term used in the statistical gamma family of distributions that has the term "constrained gamma". The mu-lambda relation is an empirically derived based on measured DSDs. Since the measured DSDs are statistical (i.e. the parameters such as Dm can be treated as statistical) the mu-lambda is not a deterministic relation.*

Answer

To clarify, the following sentence has been added to the paper:
"When an empirical relation between shape and scale parameters is used the model is often called constrained-gamma. Note that the term "constrained-gamma" denote a gamma DSD model in which the shape and rate parameters are linked by a deterministic function. Mathematically, this is equivalent to reducing the number of free parameters from three to two, which is convenient in radar-based DSD retrievals. However, the uncertainty related to estimating μ and Λ based on observed DSD spectra remains. Hence the constrained gamma DSD model and all its associated moments still remains stochastic in nature."

**   Line 12: Sentence beginning 'The most difficult ..' This is true of all retrievals of the DSD and R. It is not surprising that NT which is 0th moment of the DSD cannot be estimated accurately using higher order moments like Z=f(M6) and Dm=M4/M3.*

Answer

Yes, indeed, it is intrinsically hard to retrieve low order moments such as N_T from higher order moments such as Z. That, combined with the fact that there is some error propagation in the retrieval procedure (i.e., N_T is the last parameter to be retrieved) makes it very challenging to get accurate estimates of $N_{T.}$

* *Abstract, Last sentence: this increase in correlation from 0.12 to 0.24 is not a meaningful increase...the scatter still looks "random"*

<u>Answer</u>

Thanks for your comment. We modified the sentence as follows:
"After careful data filtering and removal of problematic Z_{hh}/Z_{dr} pairs, the correlation coefficient for the retrieved N_T values remained low, only slightly increasing from 0.12 into 0.24."

* *Line 33: Surely by now the entire DSD community is aware that N0-mu relation is not physical.*

<u>Answer</u>

Noted. But it does not hurt to repeat it and provides some useful context to young career scientists who just started working on the topic. It may also be useful to people who are not very familiar with the theory behind drop size distributions.

* *Line 46: I do not agree that calibration offsets in Zh and Zdr are often overlooked. The US Nexrad system has done extensive work to reduce the uncertainty of Zdr to within -+0.1 dB. To this, one can add the German DWD, and MeteoFrance as well.*

<u>Answer</u>

Yes, we know that there are operational radars (like the ones you mentioned above) for which there is an in-depth procedure to monitor and correct calibration issues. However, unfortunately, this is not the case everywhere, especially for research radars. Our study highlights the importance of this issue, without minimizing the good work done by other researchers, institutes and agencies. In order to convey the right meaning, we slightly modified the corresponding text:

"Finally, one last issue that tends to be overlooked is that radar measurements are likely to contain systematic errors in the form of calibration offsets in Z_{hh} and Z_{dr}. A possible error in the latter could induce large biases in the retrieved DSDs, especially in light rain with low Z_{dr} and small signal to noise ratio. Several operational polarimetric weather radar networks such as the US Nexrad (Hubbert and Pratte, 2007) and the German DWD network (Frech and Hubbert, 2020) have already devoted extensive efforts toward mitigating these calibration issues. However, achieving and maintaining good calibration over time for research radars remains challenging."

* *Line 71: The instrument does not possess the resolution to measure the drizzle and very small drops. This is also termed as truncation of the DSD and the shape factor will be biased to strongly positive values with convex down shape at the small drop end.*

Answer

Thank you for your comment. As we stated in text, we are perfectly aware of the limitations of the Parsivel. The modified sentence (please see below) now clearly mentions that Parsivel has difficulties measuring small drops:

"The working principle, strengths and limitations of the $PARSIVEL^2$ have already been discussed in great depth in previous studies and will not be part of this study (Löffler-Mang and Joss, 2000; Tokay et al., 2014; Battaglia et al., 2010; Thurai et al., 2011, Raupach and Berne 2015;). For example, the Parsivel is susceptible to errors in the lower drop diameter range which can affect the DSD shape and number concentrations. However, no efforts have been done to try to correct for these issues within the context of this study."

* *Line 85: "comparable" is not the correct description.... you are only sampling in time to get 30 s sampling.*

Answer

Thank you for the comment. Yes, strictly speaking they are not comparable. However, since our only option is to compare data from different sensors which have different specifications, we should at least try to first adjust them in a way in order to make them comparable to each other in a sort of a way. After we down-sampled TARA's Z_{hh} and Z_{dr} measurements over successive 30 s sampling intervals, even though the sensors are no similar, we could say that their data are kind of comparable.

Line 98: fig 1 does not appear to have a clear melting layer....what is mean by clear? the vertical streaks of Z above the BB indicates vertical air motion.

&

Line 112: the BB does not look steady, rather the vertical streaks in Z well above the BB depict some vertical air motion.

<u>Answer</u>

Thanks for your comment. Yes, indeed vertical streaks of reflectivity above the bright band indicate some vertical air motion. However, the classification into stratiform and convective should not be taken too strictly as events are likely to contain a mixture of different rain types. To clarify this point, the text has been modified to:
"1. Each event must consist of predominantly stratiform rain and exhibit a well-defined melting layer signal in the radar data."

Eq. 1: the use of NT was introduced by Chandrasekar and Bringi to emphasie that NT = 0th moment =total number density which makes this form similar to what statisticians would use.

<u>Answer</u>

Noted. We added the following reference in the text:

- Bringi, V. N., and V. Chandrasekar, 2001: Polarimetric Doppler Weather Radar: Principles and Applications. Cambridge Uni- versity Press, 636 pp.

*	*Line 154: "empirical" or "statistical"?*

Answer

Done. Empirical relationship.

*	*Eq. 7: is there any physical basis for this power law?*

Answer

Thank you for your comment. We re-arranged Section 3 and now Subsection 3.4 is before Subsection 3.3 where we clarify the reason behind our model choice. As stated in the manuscript the power law model is easier to physically justified rather than a parabola.

*	*Line 163: Dmax is approximately 3*Dm...see Carey and Petersen*

Answer

Thanks for the comment. Yes, indeed, we modified the sentence as follows: "where σ_{hh} (mm^2) and σ_{vv} (mm^2) are the copolar radar cross-sections of raindrops with equivolume spherical diameter D, at horizontal and vertical polarization, respectively, and D_{max} (mm) is a reasonable maximum drop diameter (e.g., 7 mm in our case). In the literature several studies tried to link D_{max} with D_0 such as Ulbrich and Atlas (1984), who concluded that $D_{max} / D_0 > 2.5$ is what is typically observed in natural rainfall, or Carey and Petersen (2015) who recommended using $D_{max} = 3 * D_0$."

*	*Line 195: The critical aspect is that Parsivel cannot measure the drizzle or smalll drops with sufficient resolution causing truncation. This causes Dm to increase as well as the decrease in the the spectral width sigma... casing mu to decease.*

Answer

Yes, this is true. We added a sentence to highlight the effect of truncation to the DSD shape itself.

"Critical aspects that were investigated are whether the μ-Λ relation remains stable with respect to different sampling resolutions, drop number concentrations, types of stratiform rain events or the validity of the gamma DSD hypothesis itself. At the same time, one has to keep in mind that the limitation of the Parsivel in terms of detection of small droplets might lead to overestimated D_m and μ values, since the width of the distribution will be underestimated."

* *Also, the stability of mu-lambda relation itself is not in question since it can be stable for the wrong reason.*

<u>Answer</u>

This does not make any sense. Why would the μ-Λ be stable for the wrong reason? And what is the link with the previous paragraph/comment?

* *Line 235: The NT is the same as M0 ie the total number density. It is not possible to estimate it from the higher order moments such as Nw or Dm. In fact the variability in NT of the DSD is larger than that of Dm or mu. This is termed as number controlled DSDs.*

<u>Answer</u>

Thanks for the comment. The fact that DSDs are predominantly number or size-controlled does not really play a crucial role in the retrieval algorithm itself. However, it is possible that the rainfall regime (i.e., number vs size-controlled DSDs) and associated scaling laws could influence the stability of the μ-Λ relationship. This is interesting but would have to be investigated in a separate paper, as it is clearly outside the scope of this study and would require new data for convective events as well. An additional sentence has been added to the text to clarify this point:

"It would be interesting to investigate whether the events for which the DSD is predominantly number controlled lead to more/less stable μ-Λ relationships than events with size-controlled DSDs."

*   *Last sentence in 5.1.3: this is known as the point-to-area or non-uniform beam filling problem. This is very well known and has been addressed by several publications*

Answer

Thank you for your comment. We added the following reference in the text:

- Ryzhkov, A. V. (2007). The Impact of Beam Broadening on the Quality of Radar Polarimetric Data, Journal of Atmospheric and Oceanic Technology, 24(5), 729-744.
- S. L. Durden and S. Tanelli, "Predicted Effects of Nonuniform Beam Filling on GPM Radar Data," in IEEE Geoscience and Remote Sensing Letters, vol. 5, no. 2, pp. 308-310, April 2008, doi: 10.1109/LGRS.2008.916068.

*   *Last sentence, 5.2: This is not surprising since NT is the M0th moment whereas Nw, Dm are of much higher order.*

Answer

Yes, indeed. See our response to previous, similar comments.

*   *Line 405: no surprise here...unless one can measure M1, M2, there is no way to improve the estimate NT.*

Answer

Thank you for the comment but we do not really agree with this. The estimation of N_T is possible without M1 and M2. If we assume that the DSDs are perfectly gamma, that Z_{hh} and Z_{dr} are perfectly calibrated, and that the μ-Λ relationship is valid, the N_T could in principle be estimated. The problem is that we have large measurement and modeling uncertainty. But there are plenty of ways to improve, for example by applying bias corrections (over time), and also, potentially, by adapting the μ-Λ relationship over time.

*	Line 447: The method of improving the correlation coeff especially for NT does not improve at al ...the corr~ 0.*

Answer

Done. See our response to previous, similar comments.