

This study examines the effect of particle inertia in forward modelling vertically pointing cloud radar Doppler spectrum. The authors have carried out theoretical analysis and validated their method with field observations. The logic of this manuscript is clear, and the research question is sound. Different approaches are clearly compared in good-quality figures. Relevant references have been cited.

However, there is one major flaw regarding the validation. Please see my comments below. In addition, the description of the new simulator is difficult to follow. Therefore, my recommendation is major revision.

Response: We want to thank the reviewer's comments and suggestions. We have modified the validation and the methodology section in the revised manuscript. The detailed responses can be seen below.

Major comments

Description of the new spectrum simulator is not clear to me. Since it is a new method, a detailed and explicit explanation is needed. I am wondering where the term is quantifying the turbulence. Also, Eq.12. looks the same as eq.9.

Response: We have rephrased the description of the simulator in the revised manuscript, please refer to section 3.2 for more details.

The validation part is questionable to me. The broadening effect seems to be exaggerated. If I understood correctly, the authors assume no horizontal and shear winds. Then, eq 5 in Borque 2016 changes to $\sigma^2 = \sigma d^2 + \sigma t^2$. σ^2 can be estimated from the observed spectrum, and the magnitude of σt^2 depends on σd^2 . If σd^2 as retrieved from the surface DSD is underestimated, σt^2 will be overestimated. Then, the broadening effect will be unrealistically large. In a word, the accuracy of σt^2 depends on how well the raindrop spectrum was constructed from surface observations. As far as I could image, the fitting process may lead to the underestimation of σd^2 . I believe the authors should carefully quantify the uncertainty of σd^2 in the revised manuscript.

In addition, in Figure 5, what is the height of the observed spectrum? How well the DSD observed at surface can be used to simulate the spectrum aloft observed by a W band radar? In other applications, these two issues do not significantly contribute to retrieval errors. Given the change of DSD can significantly affect the evaluation results, I am afraid they should be well discussed in this study.

Response: We want to thank the review's comments. In the revised manuscript we directly utilized the observed DSD from the disdrometer to simulate the Doppler spectrum instead of using the Marshall-Palmer fitting function as in the previous manuscript. This change is intended to eliminate the DSD error caused by the fitting process. In the revised manuscript we highlight that the Doppler spectrum comparison shown in section4 is not used for validation purpose but as an illustrative example. We made a thorough discussion on the representative of the surface-observed

DSD for the W-band radar observation and the uncertainty of the σ_t estimation in the revised manuscript:

Line 387: "...The observed DSD is shown in Figure 6a, and the corresponding WACR-observed Doppler spectrum is shown as the black line in Figure 6b. Based on the observed DSD, the radar Doppler spectrum for the droplets falling in still air is generated (not shown), from which the DSD-contributed Doppler spectrum width (σ_D) is estimated as 1.34 ms^{-1} . Since the wind shear broadening contribution (σ_S) to radar Doppler spectrum is generally smaller than σ_D and the turbulence broadening (σ_t) (Borque, Luke et al. 2016), here we neglect the σ_S contribution and estimate σ_t as:

$$\sigma_t^2 = \sigma_O^2 - \sigma_D^2$$

Where σ_O is the observed Doppler spectrum width, which is 1.46 ms^{-1} in this example, and σ_t is estimated as 0.58 ms^{-1} . To estimate the accuracy of σ_t , we further assume that the observed DSD is the only source of the uncertainty. Considering that the accuracy of the droplets size measurement of the disdrometer is approximately $\pm 5\%$ (Wang, Bartholomew et al. 2021), the uncertainty of σ_D and σ_t is estimated as 0.15 ms^{-1}

..."

Line 419: "...The purpose of the Doppler spectrum comparison is not for a robust validation but used as an illustrative example to show the morphology of the simulated Doppler spectrum in real environment and to discuss the required measurements would be used for robust Doppler spectrum simulator validation. To a certain degree, a more consistency Doppler spectrum morphology is identified between the observation and from the PBS simulator, especially for the right edge of the spectrum. However, great cautions should be taken for further interpretation as both of the simulators cannot represent the left part of the Doppler spectrum and the second notches very well. This discrepancy is mainly because the observed DSD by disdrometer may not an adequate representation of the hydrometeors that contribute the Doppler spectrum observed by WACR. Specifically, there are three critical challenging issues should be overcome before a solid and convincing Doppler spectrum simulator evaluation effort being performed: 1) the disdrometer is located at the surface, while the lowest measurement height of WACR is 460m. When the rain droplets fall, droplets may collide, breakup, and being advected from adjacent region by the horizontal wind; Thus a large uncertainty is expected to use the surface-observed DSD to represent the hydrometeor distribution at 450m above; 2) the observed DSD from the disdrometer only measure droplets with 20 size categories, which is insufficient for the physics-based simulation to generate a smooth and complete Doppler spectrum; 3) the uncertainty of the estimated σ_t is challenging to be well constrained due to the large uncertainty of the observed DSD mentioned above. A comprehensive and solid validation of the Doppler spectrum simulator require simultaneous and well- aligned DSD and Doppler spectrum measurement, large number of the measured droplet size categories and carefully estimation of the measurement; large number of the measured droplet size categories and carefully estimation of the environment turbulence broadening factors.

Technical issues

I have some suggestions for technical corrections, but I am not a native speaker.

Response: we appreciate the reviewer's edits. All the suggested corrections have been made in the revised manuscript.

L22. consistent with

L27. applications for cloud/precipitation

L28. microphysical and dynamical

L34. For a vertical

L35. Provide

Either using positive or negative to indicate downward is fine, but it is appreciated to make a statement in each figure's caption.

Response: we appreciate the reviewer's suggestions. Changes have been made in the caption of Figure 5 and figure 6.

L53. and many other places. Spectral broadening is contributed by a list of factors such as turbulence, horizontal wind, spectral window etc. In some cases, turbulence dominates this broadening effect.

Response: We want to thank the reviewer's comments. This sentence (and may other places) has been modified in the revised manuscript:

Line 53: "...More specifically, the Doppler spectrum width is mainly contributed by the spread of the still-air hydrometeor terminal velocity, the horizontal and vertical wind shear within the radar observation volume, and the environment turbulence..."

Line 390: "...Since the wind shear broadening contribution (σ_s) to radar Doppler spectrum is generally smaller than σ_D and the turbulence broadening (σ_t) (Borque, Luke et al. 2016), here we neglect the σ_s contribution and estimate σ_t as..."

L162. This work is published on a journal with which not many cloud radar people familiar, please detail this method.

Response: In the revised manuscript we briefly introduced the turbulent wind generation method and cited the codes we used in this study.

Line 172: "...In this study we adapt the approach proposed by Deodatis (1996) by using the Spectral Representation Method (SRM) to generate the turbulent wind field based on a predefined

Von Karman energy spectrum. The SRM is widely used in the wind engineering community due to its high accuracy, simplicity, and computational efficiency. (Shinozuka and Deodatis 1991, Zhao, Huang et al. 2021). Here, the 1-D turbulence wind is generated with 2 Hz sampling frequency, 1000s duration and with standard deviation of 0.3 ms^{-1} , the codes being applied to generate the wind can be accessed from Cheynet (2020)..."

L164-166. This sentence is confusing. Spectrum width is affected by hydrometeor size distribution, how can it be a constant value?

Response: Here we did a theoretical estimation of the turbulence broadening term (σ_t) by assuming the Doppler spectrum width is only broadened by turbulence. We have added the equation being applied to estimate σ_t . The sentence has been modified in the revised manuscript:

Line 178: "...The selection of 0.3 ms^{-1} standard deviation is based on a quantitatively estimation of cloud radar observation under a typical cloudy environment. Specifically, for the convective cloud system with eddy dissipation rate (ϵ) of $5 \times 10^{-3} \text{ m}^2 \text{ s}^{-3}$ (Mages, Kollias et al. 2022), the turbulence-contributed Doppler spectrum width (σ_t) from a vertical pointing radar with 30m range resolution(ΔR) and 0.3° beamwidth (θ) at 1km height is estimated to be 0.27 ms^{-1} based on the equation from Borque, Luke et al. (2016):

$$\epsilon \approx \frac{\sigma_t^3}{\sigma_z(1.35\alpha)^{3/2}} \left(\frac{11}{15} + \frac{4}{15} z^2 \frac{\sigma_x^2}{\sigma_z^2} \right)^{-3/2} \quad (1)$$

Where α is the Kolmogorov constant with 0.5, $\sigma_z = 0.35 * \Delta R$, $\sigma_x = \frac{\theta}{4\sqrt{\ln 2}}$, θ is the one-way half-power width with unit of radian. z is height above surface.
..."

L188. close.

Eq.13. where is n in St?

Response: n represents each simulation step.

L292. L306, and many other places. Turbulent environment

L306. echo?

L307. of

L338. spectra from two approaches are consistent with each other.

L340. add a comma before but

L343. approaches

L344. The black line

Figure4. $\text{dB}(10\log_{10}(\text{mm}^6 \text{m}^{-3}))$

Response: Changes have been made in the updated figure.

Figure4 and many other places. Simulated approach looks strange to me. I would call it physics-based approach.

Response: We have renamed the proposed method as Physics-Based Simulation (PBS) approach.

Line 338: "...the broadening of the right edge of the radar Doppler spectrum from the physics-based simulation (PBS) approach..."

L4225. Can be employed in more studies

Reference

Borque, P., et al. (2016). "On the unified estimation of turbulence eddy dissipation rate using Doppler cloud radars and lidars." Journal of Geophysical Research: Atmospheres **121**(10): 5972-5989.

Cheyne, E. (2020). Wind field simulation (text-based input), Zenodo, Tech. Rep., 2020, doi: 10.5281/ZENODO.3774136.

Deodatis, G. (1996). "Simulation of ergodic multivariate stochastic processes." Journal of engineering mechanics **122**(8): 778-787.

Mages, Z., et al. (2022). "Surface-based observations of cold-air outbreak clouds during the COMBLE field campaign." Atmospheric Chemistry and Physics Discussions: 1-39.

Shinozuka, M. and G. Deodatis (1991). "Simulation of stochastic processes by spectral representation."

Wang, D., et al. (2021). Analysis of Three Types of Collocated Disdrometer Measurements at the ARM Southern Great Plains Observatory, Oak Ridge National Lab.(ORNL), Oak Ridge, TN (United States). Atmospheric ...

Zhao, N., et al. (2021). "Simulation of ergodic multivariate stochastic processes: An enhanced spectral representation method." Mechanical Systems and Signal Processing **161**: 107949.