

1 **Particle Inertia Effects on Radar Doppler Spectra Simulation**

2 Zeen Zhu¹, Pavlos Kollias^{1,2} and Fan Yang¹

3 ¹Environmental and Climate Sciences Dept, Brookhaven National Laboratory, Upton, NY, USA

4 ² Division of Atmospheric Sciences, Stony Brook University, Stony Brook, NY, USA

5 *Correspondence to: Zeen Zhu (zzhu1@bnl.gov)*

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7 **Abstract.** Radar Doppler spectra observations provide a wealth of information about cloud and
8 precipitation microphysics and dynamics. The interpretation of these measurements depends on
9 our ability to simulate these observations accurately forward. The effect of small-scale turbulence
10 on the radar Doppler spectra shape has been traditionally treated by implementing the convolution
11 process on the hydrometeor reflectivity spectrum and environment turbulence. This approach
12 assumes that all the particles in the radar sampling volume respond the same to turbulent scale
13 velocity fluctuations and neglects the particle inertial effect. Here, we investigate the inertia effects
14 of liquid phase particles on the forward modelled radar Doppler spectra. A physics-based
15 simulation is developed to demonstrate that big droplets, with large inertia, are unable to follow
16 the rapid change of velocity field in a turbulent environment. These findings are incorporated to a
17 new radar Doppler spectra simulator. Comparison between the traditional and the newly
18 formulated radar Doppler spectra simulators indicates that the conventional simulator leads to an
19 unrealistic broadening of the spectrum, especially in strong turbulence environment. This study
20 provides clear evidence to illustrate the droplets inertial effect on radar Doppler spectrum and
21 develops a physics-based simulator framework to accurately emulate the Doppler spectrum for a
22 given Droplet Size Distribution in turbulence field. The proposed simulator has various potential
23 applications for the cloud/precipitation studies and provides a valuable tool to decode the cloud
24 microphysical and dynamical properties from Doppler radar observation.

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32 **1 Introduction**

33 The radar Doppler spectrum represents the frequency (velocity) distribution of the
34 backscattered radar signal at a particular range. For a vertically pointing radar, the Doppler
35 spectrum provides the distribution of the backscattered signal over a range of Doppler velocities,
36 whose value depends on the dynamical (i.e., vertical air motion) and cloud microphysical (i.e.,
37 hydrometeors concentration and sizes) properties within the radar sampling volume. A variety of
38 research applications that utilize the full radar Doppler spectrum have been developed. For
39 instance, Doppler spectrum can be used to retrieve rain Droplet Size Distribution (DSD) (Atlas et
40 al., 1973), remove clutters and identify hydrometeor signals (Williams et al., 2018; Luke et al.,
41 2008; Moisseev and Chandrasekar, 2009), identify drizzle development stage (Zhu et al.,
42 2022; Acquistapace et al., 2019), retrieve vertical air motion (Kollias et al., 2002; Williams,
43 2012; Zhu et al., 2021), characterize the melting-layer properties (Li and Moisseev, 2020; Mróz et
44 al., 2021), and to improve the representation of cloud microphysical process in model (Kollias et
45 al., 2011b). Combined with the depolarization capability, Doppler spectrum can also be used for
46 cloud-phase classifications and to investigate ice-cloud microphysical process (Luke et al.,
47 2010; Luke et al., 2021; Kalesse et al., 2016; Oue et al., 2018). The forward Doppler spectra
48 simulator can further be utilized to simulate radar observation from the modeling output to evaluate
49 the model performance (Oue et al., 2020; Mech et al., 2020; Silber et al., 2022). The list of widely
50 application of the Doppler spectrum in the cloud-precipitation research mentioned above is by no
51 means exhaustive.

52 Despite the extensive applications, an unambiguous interpretation of radar Doppler
53 spectrum still remains a challenging task in the cloud radar community. One important reason is a
54 lack of full understanding of the entanglement between the hydrometeor microphysics and
55 environment dynamics as well as their manifestation on the Doppler spectrum morphology
56 (Kollias et al., 2002). More specifically, the Doppler spectrum width is mainly contributed by the
57 spread of the still-air hydrometeor terminal velocity, the horizontal and vertical wind shear within
58 the radar observation volume and the environment turbulence; while the Doppler frequency shift
59 is a combined measure of the air motion and the particles falling velocity (Doviak, 2006). A
60 successful separation of the microphysical and dynamical contributions to Doppler spectrum is
61 essential to reduce retrieval uncertainties and to better characterize the cloud-precipitation
62 properties (Zhu et al., 2021).

63 Doppler spectrum simulators have been invaluable for the interpretation of the radar
64 Doppler spectrum shape (Capsoni et al., 2001;Oue et al., 2020;Kollias et al., 2011a;Maahn et al.,
65 2015). Traditionally, the impact of turbulence on the shape of the radar Doppler spectrum is
66 represented by the convolution of the still air (no air motion) hydrometeor reflectivity spectrum
67 with a Gaussian distribution (Gossard and Strauch, 1989). The width of the Gaussian distribution
68 is parameterized as a function of the radar parameters and the turbulence intensity often
69 represented in terms of eddy dissipation rate (Borque et al., 2016). This approach is only valid
70 under the assumption that the droplet inertia effect is negligible and droplets with different sizes
71 can follow exactly the environment wind field. In reality, however, big droplets with large inertia
72 cannot follow the rapid change of wind velocity field unlike small droplets perform (Yanovsky,
73 1996;Lhermitte, 2002). Not accounting for the particle inertia effect can lead to a misinterpretation
74 of the Doppler spectrum and cause large uncertainties for retrieval products (Nijhuis et al., 2016).

75 Several physics-based frameworks have been proposed to simulate the droplet motions in
76 turbulence field (Khvorostyanov and Curry, 2005;Lhermitte, 2002). Here, the approach proposed
77 by Lhermitte (2002) is used to illustrate the droplets inertial effect and to investigate this effect on
78 the radar Doppler spectrum. In detail, we aim to answer the following questions: 1) How does
79 inertia affect the response of a droplet in a fluctuating turbulent wind field? 2) Is this effect
80 significant on the simulated and observed radar Doppler spectrum? and 3) How can we account
81 for the droplet inertia in radar Doppler spectrum simulators? Building on these investigations, a
82 new approach to generate radar Doppler spectrum is described.

83 The structure of this paper is organized as follows: section 2 describes the physical
84 modeling framework used to simulate the liquid droplet motion and to illustrate the droplets inertia
85 effect in a turbulent environment; section 3 proposes the physics-based Doppler spectrum
86 simulator and compares the emulated spectra to the ones generated from the traditional method;
87 in section 4 one observed Doppler spectrum is used as an illustrative example to compare the
88 Doppler spectrum generated from the two simulators; section 5 concludes the major results of this
89 study and followed by a discussion.

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94 2 Droplets inertial effect in a turbulent environment

95 In this section, a physics-based simulation framework used to illustrate the droplets inertia
96 effect in a turbulent environment is presented. First, we will introduce the equations used to
97 describe the velocity of droplets moving in the air. Then a generated turbulent wind field is applied
98 to the simulation framework to illustrate the droplet inertial effect and the potential implication on
99 the generated Doppler spectrum.

100

101 2.1 Motion of droplets in the air

102 The fundamental dynamical framework of describing the droplets motion in the air is
103 adapted from Lhermitte (2002), p.81. Assuming a liquid droplet with diameter of D , the motion
104 of the droplet in the air can be described as:

105

$$F - mg = m \frac{dV_D}{dt} \quad (1)$$

106

107 where m is the droplet mass, V_D is the droplet velocity, F is the drag force exerted by wind
108 expressed as:

109

$$F = \frac{C_d S (V_w - V_D)^2 \rho_a}{2} \cdot \text{sgn}(V_w - V_D) \quad (2)$$

110 Where C_d is the wind drag coefficient, ρ_a is air density, S is the droplet cross section normal to
111 wind direction. V_w is wind velocity and $(V_w - V_D)$ indicates droplet velocity with respect to air.
112 In a turbulent environment, V_w could be either positive or negative, thus the exerted wind can either
113 accelerate or decelerate the droplet velocity. To this end, the sign function $\text{sgn}(V_w - V_D)$ is
114 included to account for the wind drag force direction.

115 For spherical droplets, S can be calculated as:

116

$$S = \frac{\pi D^2}{4} \quad (3)$$

117

118 and droplet mass (m) is calculated as:

119

$$m = \frac{1}{6} \pi \rho_l D^3 \quad (4)$$

120 where ρ_l is liquid water density.

121 The only unknown factor is the drag coefficient C_d , which should be derived from
 122 experiment. Numerous studies have been conducted to measure the sphere terminal velocity in
 123 fluid and estimate C_d as a function of Reynolds number (Re) (Schlichting and Kestin, 1961;Lapple
 124 and Shepherd, 1940;Haider and Levenspiel, 1989). However, the derived $C_d - Re$ relationships in
 125 the previous studies are applied for rigid spherical particles. For the rain droplets with large
 126 diameter, the droplet is distorted and the exerted drag coefficient for a given Re deviates from the
 127 rigid sphere. To this end, the drag term of the rain droplet is obtained from the measurement of
 128 the terminal velocity of liquid droplets. Here, we adapt the experiment data from Gunn and Kinzer
 129 (1949), in which study C_d and Re are estimated for liquid droplets with diameter ranging from 100
 130 μm to 5.8 mm . The experiment-derived C_d and Re are shown in Figure 1, we further fit the data
 131 with a fifth-degree polynomial (red line) to estimate C_d for a given Re :

$$132 \quad \log C_d = 1.4277 - 0.8598 \times \log Re + 0.0699 \times (\log Re)^2 - 0.0023 \times (\log Re)^3 - \quad (5)$$

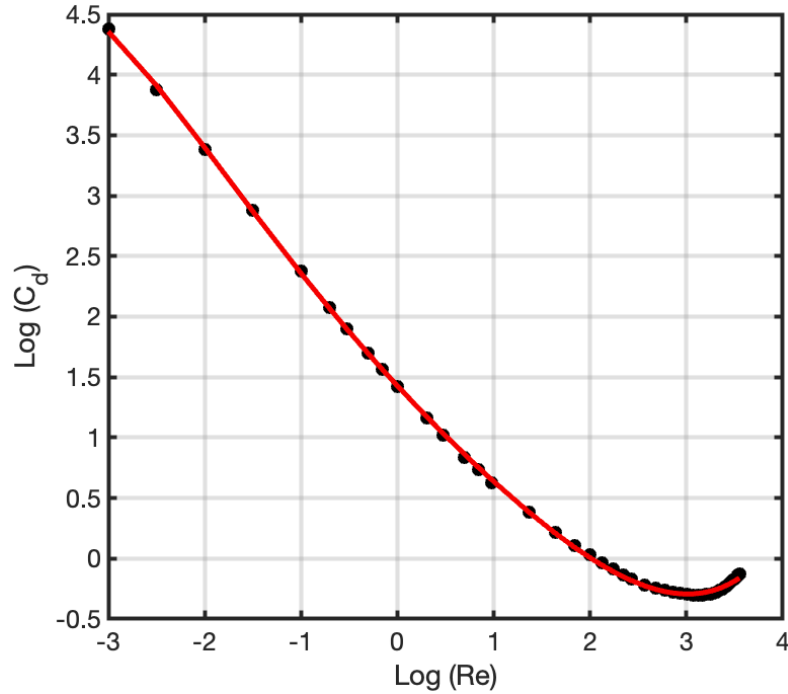
$$133 \quad 0.0003 \times (\log Re)^4 + 0.0013 \times (\log Re)^5$$

134 Where the Reynolds number Re is represented as:

$$135 \quad Re = \frac{|V_w - V_D| D \rho_a}{\mu} \quad (6)$$

136 where μ is the air dynamic viscosity. The values used for ρ_a , ρ_l , and μ are 1.22 kg m^{-3} ,
 137 1000 kg m^{-3} , $1.81 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$, corresponding to atmospheric environment of 15°C and
 138 1000 hPa .

139 Combining (1)-(6), a set of ordinary differential equation is constructed, the droplet velocity (V_D)
 140 for a given droplet with diameter D as a function of time can be resolved numerically for a given
 141 wind field (V_w).
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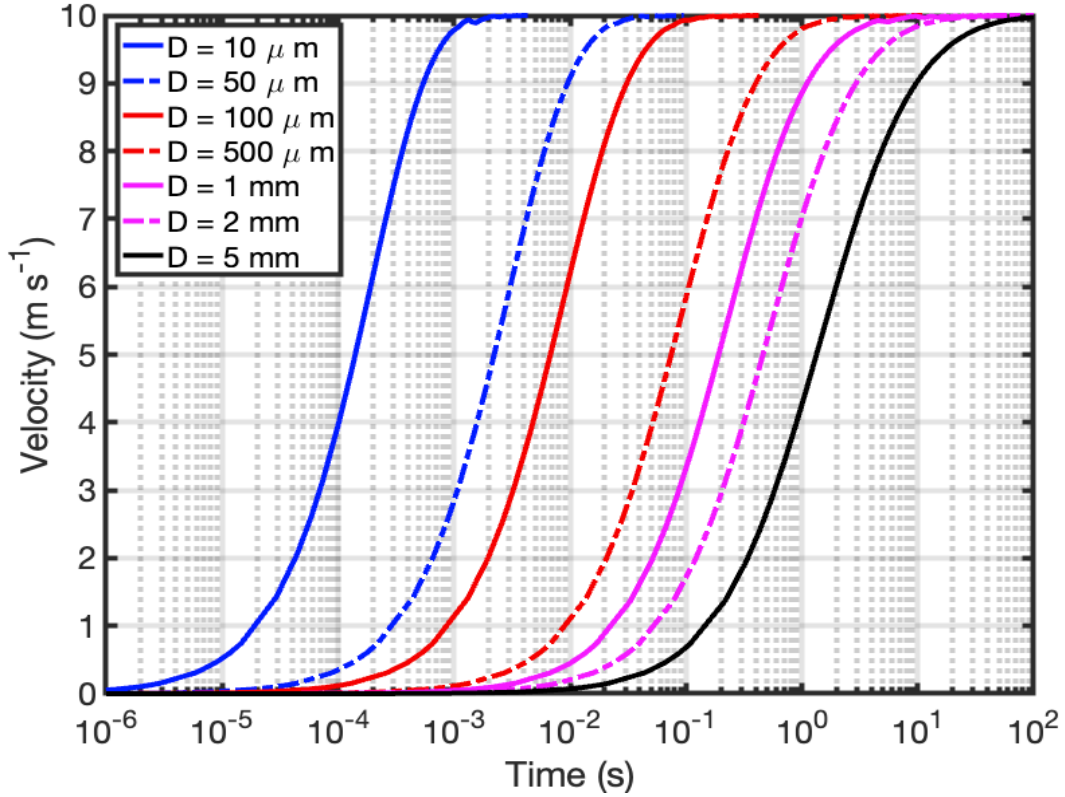
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149 Figure 1: The black dots represent the experiment-derived C_d and Re adapted from Gunn and
 150 Kinzer (1949). The red line is a fifth-degree polynomial fitting function.

151

152 2.2 Illustration of droplet inertial effect

153 We first illustrate the inertial effect by calculating droplets motion using a constant wind
 154 velocity. For simplicity, here we assume all the droplets are moving horizontally, thus the gravity
 155 (mg) is neglected in Eq.1. Seven droplets with diameters of $10\ \mu m$, $50\ \mu m$, $100\ \mu m$, $500\ \mu m$, 1
 156 mm , $2\ mm$, $5\ mm$ are selected to cover the size range of cloud droplet, drizzle and raindrops.
 157 Initial velocity of all the droplets is $0\ ms^{-1}$, a constant wind velocity with $10\ ms^{-1}$ is exerted upon
 158 the droplets when $t > 0\ s$. Due to the wind drag force, droplets start to move but with different
 159 accelerations depending on droplet inertia: droplets with small inertia are accelerated more quickly
 160 than larger ones. This effect is clearly illustrated in Figure 2: droplet with diameter of $10\ \mu m$
 161 quickly reach to the wind velocity within only $0.002s$, while droplets with $1\ mm$ and $5\ mm$ need
 162 5 and $50s$ to adjust their motion to the exerted wind velocity. The different response time of
 163 droplets with different sizes to the exerted wind velocity suggests that small droplets are more
 164 capable to follow the velocity variation than their large counterparts.



165

166 Figure 2. Velocity of droplets with diameter of $10 \mu\text{m}$ (blue solid line), $50 \mu\text{m}$ (blue dash-dot line),
 167 $100 \mu\text{m}$ (red line), $500 \mu\text{m}$ (red dash-dot line), 1 mm (magenta solid line), 2 mm (magenta dash-
 168 dot line) and 5 mm (black solid line) as function of time after exerted by a constant wind with 10
 169 ms^{-1} velocity.

170

In real atmosphere, air velocity is not constant but fluctuates with time as a representative
 171 of turbulent nature. In this study we adapt the approach proposed by Deodatis (1996) by using the
 172 Spectral Representation Method (SRM) to generate the turbulent wind field based on a predefined
 173 Von Karman energy spectrum. The SRM is widely used in the wind engineering community due
 174 to its high accuracy, simplicity and computational efficiency. (Shinozuka and Deodatis, 1991; Zhao
 175 et al., 2021). Here, the 1-D turbulence wind is generated with 2 Hz sampling frequency, 1000s
 176 duration and with standard deviation of 0.3 ms^{-1} , the codes being applied to generate the wind can
 177 be accessed from Cheynet (2020). The selection of 0.3 ms^{-1} standard deviation is based on a
 178 quantitatively estimation of cloud radar observation under a typical cloudy environment.
 179 Specifically, for the convective cloud system with eddy dissipation rate (ε) of $5 \times 10^{-3} \text{ m}^2 \text{ s}^{-3}$
 180 (Mages et al., 2022), the turbulence-contributed Doppler spectrum width (σ_t) from a vertical

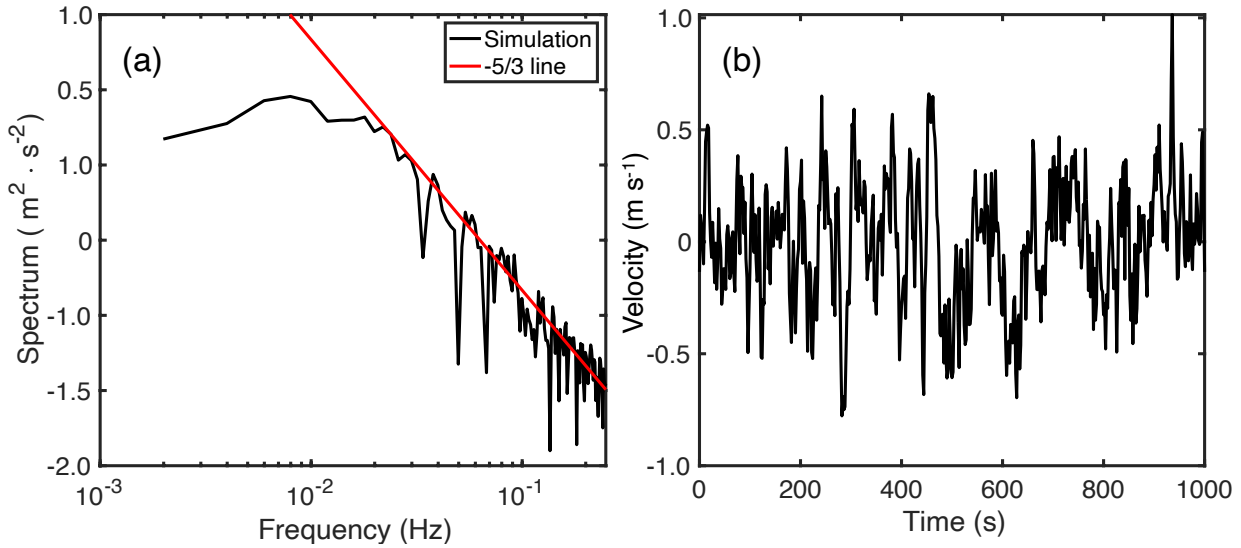
181 pointing radar with 30m range resolution(ΔR) and 0.3° beamwidth (θ) at 1km height is estimated
 182 to be 0.27 ms^{-1} based on the equation from Borque et al. (2016):

183

$$184 \quad \varepsilon \approx \frac{\sigma_t^3}{\sigma_z(1.35\alpha)^{3/2}} \left(\frac{11}{15} + \frac{4}{15} z^2 \frac{\sigma_x^2}{\sigma_z^2} \right)^{-3/2} \quad (7)$$

185 Where α is the Kolmogorov constant with 0.5, $\sigma_z = 0.35 * \Delta R$, $\sigma_x = \frac{\theta}{4\sqrt{\ln 2}}$, θ is the one-way
 186 half-power width with unit of radian. z is height above surface.

187 The spectrum and time series of the generated air velocity are shown in Figure 3: the
 188 turbulence spectrum (Figure 3a) characterizes typical inertial subrange of the turbulence scale with
 189 a standard deviation of 0.3 ms^{-1} (Figure 3b).



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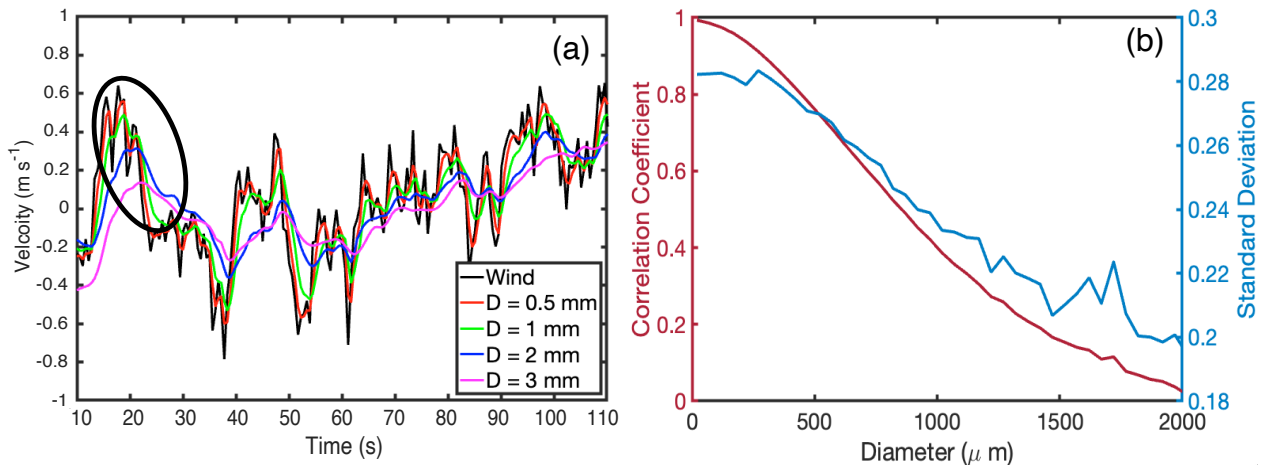
191 Figure 3. (a) Spectrum of the simulated turbulence (black line), red line represents the -5/3 slope.
 192 (b): Time series of vertical velocity for the simulated turbulence.

193

194 The generated air velocity is assigned to V_w in Eq. (2) to simulate the motion of droplets
 195 with initial velocity set as 0 ms^{-1} . Figure 4a shows the time-dependend velocity of droplets with
 196 selected diameter of 0.5 mm , 1 mm , 2 mm , 3 mm . Droplets with different sizes response
 197 differently with the change of wind velocity, and there are two notable characteristics due to the
 198 inertial effect (highlighted in the black oval in Fig. 4a). First, large droplets need longer time to
 199 adjust to the wind velocity, thus there is a distinct time-lag when the peak velocity is reached for
 200 different particles. Second, in addition to the time-lag, the peak velocity reached by the large

201 droplets is smaller than the small droplets. Here, we use correlation coefficient between the actual
 202 wind velocity and the droplet velocity to quantify the inertial effect. A correlation coefficient of 1
 203 represents droplets can follow exactly the wind velocity and a correlation coefficient less than 1
 204 indicates a time-lag effect between the wind and droplet velocity due to droplet inertia. Figure 4b
 205 shows that the correlation coefficient is close to 1 when the droplets are smaller than 50 μm but it
 206 decreases dramatically as droplet size increases. The correlation coefficient reaches to 0 when
 207 diameter reaches to 2000 μm . In addition, for droplets with diameters smaller than 300 μm the
 208 standard deviation of the actual droplet velocity is 0.29 ms^{-1} (blue curve, Figure 4b), which is close
 209 to standard deviation of the background wind field (0.3 ms^{-1}). As droplet size increases, the
 210 velocity variation decreases due to droplet inertial effect.

211 The simulation results shown in Figure 4 suggest that droplets with diameter smaller than
 212 300 μm are less affected by inertia and can quickly adjust their velocity to the imposing wind field,
 213 and thus, small cloud droplets can be treated as perfect air tracers (Kollias et al., 2001). On the
 214 other hand, large droplets ($D > 0.5 \text{ mm}$) exhibit a time lag in their response to the air motion and
 215 an amplitude reduction (inertia-based filtering). As the observed Doppler velocity is a combined
 216 measure of the droplet velocity and the ambient air motion, this droplet inertial effect is expected
 217 to have a considerable effect on the generated radar Doppler spectrum. In the following section,
 218 we will illustrate how the radar Doppler spectrum is affected by droplet inertia and how to account
 219 for this effect using a new radar Doppler spectrum simulator.



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223 Figure 4. (a) Generated wind velocity field (black line) and the simulated velocity for particles
 224 with diameter of 0.5mm (red line), 1mm (green line), 2mm (blue line) and 3mm (magenta line)
 225 from 10s to 110s. The black oval indicates the period showing droplet inertia effect. (b) Left axis:
 226 correlation coefficient between wind field and droplet velocity for different droplets size; right
 227 axis: standard deviation of the droplets velocity with different droplets size. Only droplets with
 228 size from 0 μm to 2000 μm are shown for the sake of clarity.

229

230 3 Radar Doppler spectrum Simulator

231 Two methodologies for simulating the radar Doppler spectrum for a given DSD and
 232 turbulence conditions are used here. The first approach is the traditional one. All droplets,
 233 independent of their sizes, are assumed to have no inertial effects and thus act like perfect tracers.
 234 In this case, the radar Doppler spectrum in a turbulent environment is represented through the
 235 convolution of a Gaussian distribution and the radar Doppler spectrum in still air which is only
 236 determined by the hydrometeor DSD (Gossard, 1981; Kollias et al., 2011, Zhu et al., 2021). A
 237 brief overview of the traditional method is described in section 3.1.

238

239 3.1 Traditional Doppler spectrum simulator

240 For a given DSD described by a number concentration $N(D)$ per unit of volume in m^{-4} , the
 241 radar reflectivity $d\eta(D)$ (m^2/m^3) from particles with diameter between D to $D + dD$ can be
 242 expressed as Lhermitte, (2002, p. 228):

$$243 \quad d\eta(D) = N(D)\sigma_b(D)dD \quad (8)$$

244 where $\sigma_b(D)$ is the backscatter cross section (m^2) of a particle with diameter D in m. Mie scattering
 245 theory is used to estimate $\sigma_b(D)$. In this formulation, the radar power spectrum distribution is
 246 provided in terms of particle size. Profiling radar do not observe the radar backscatter-energy
 247 power spectrum $d\eta(D)$ but the radar Doppler spectrum density $S_q(V_t)$ where V_t is the droplet still-
 248 air terminal velocity. The conversion from droplet size to velocity requires a $V_t(D)$ relationship.
 249 Here, the function proposed by Lhermitte, (2002, p.120) is used to estimate V_t as a function of
 250 droplet diameter (D):

$$251 \quad V_t(D) = 920 \times (1 - \exp(-6.8 \times D^2 - 4.88 \times D)) \quad (9)$$

252 where the unit of D and V_t is in cm and cms^{-1} respectively. Subsequently, the radar Doppler
 253 spectral density $S_q(V_t)$ in units of $m^2m^{-3} (ms^{-1})^{-1}$ is given by:

254
$$S_q(V_t) = \frac{d\eta}{dV_t} = \frac{d\eta}{dD} \frac{dD}{dV_t} = N(D)\sigma_b(D) \frac{dD}{dV_t} \quad (10)$$

255 where $\frac{dD}{dV_t}$ is estimated from Eq. 9.

256 The $S_q(V_t)$ is the “still-air” radar Doppler spectrum where the only velocity contribution
 257 is the droplet still-air terminal velocity . In the real atmosphere, the observed velocities from the
 258 radar include the turbulent motions with scales larger or smaller than that of the radar sampling
 259 volume (Kollias et al., 2001; Borque et al., 2016). The contribution of turbulence on Doppler
 260 spectrum broadening is commonly parameterized as σ_t . It is important to note that the σ_t value
 261 also strongly depend on the radar sampling characteristics (Kollias et al., 2005). For the same EDR
 262 value, σ_t is lower for radar systems with short time dwell, narrow beamwidth and short pulse
 263 length (Borque et al., 2016). The σ_t is typically used to introduce the effect of turbulence on the
 264 radar Doppler spectrum. Under the assumption of isotropic turbulence, the distribution of the
 265 turbulent motions within the radar sampling volume can be approximated using a Gaussian
 266 function:

267
$$G(v) = \frac{1}{\sigma_t \sqrt{2\pi}} \times \exp\left(-\frac{1}{2} \left(\frac{v}{\sigma_t}\right)^2\right) \quad (11)$$

268 And its impact on radar Doppler spectrum is formulated by the convolution between $S_q(V_t)$ and
 269 $G(v)$ (Gossard and Strauch, 1989) as:

270
$$S(v) = (S_q * G)(v) = \int_{-\infty}^{\infty} S_q(u)G(v - u)du \quad (12)$$

271 3.2 Physics-simulation based Doppler spectrum simulator

272 In this approach, instead of using a Gaussian distribution to parameterize turbulence field
 273 and applying the convolution process to represent the interaction between DSD and environmental
 274 turbulence, the radar Doppler spectrum is generated using a large number of simulated droplet
 275 velocities during a given simulation period. Specifically, for droplet with diameter of D moving in
 276 a turbulent flow, the droplet velocity at each specific time can be numerically resolved as $V(D, t)$
 277 based on the ordinary differential equations described in section 2.1.

278 The radar Doppler spectrum density at each time step $S_t(v)$ can be directly estimated as:

279
$$S_t(v) = \frac{\sum N(D_{V_{i-1} \sim V_i})\sigma_b(D_{V_{i-1} \sim V_i})}{V_i - V_{i-1}} \quad (13)$$

280 Where $D_{V_{i-1} \sim V_i}$ represents the diameter of the particle with velocity within the predetermined
 281 Doppler velocity interval $[V_{i-1}, V_i]$ at each timestep, $N(D_{V_{i-1} \sim V_i})$ and $\sigma_b(D_{V_{i-1} \sim V_i})$ indicate the

282 number concentration and the backscatter power corresponding to each diameter. The
 283 predetermined Doppler velocity V_i is depended on the radar configuration of Nyquist velocity
 284 ($V_{nyquist}$) and the number of the Fast Fourier Transform points ($NFFT$):

$$285 \quad V_i = -V_{nyquist} + \frac{2V_{nyquist}}{NFFT} \times i; i = [1, 2, 3, \dots, NFFT] \quad (14)$$

286 The final Doppler spectrum can be obtained by averaging $S_t(v)$ during the simulated period:

$$287 \quad S(v) = \frac{1}{N_t} \sum_{t=1}^{t=N_t} S_t(v) \quad (15)$$

288 where N_t is the total simulation timesteps:

$$289 \quad N_t = T \times f \quad (16)$$

290 Where T and f is the simulated time and the sampling frequency of the generated turbulence
 291 wind field.

292 It is noted that the emulated radar Doppler spectrum is dependent on the generated
 293 turbulence flow, which is controlled by three parameters: time duration (T), sampling frequency (f)
 294 and standard deviation (σ). σ quantify the turbulence intensity while T and f determine the total
 295 emulated time steps. Here we use the typical cloud radar configurations to guide the choice of T
 296 and f . Specifically, T is set as 2s and f is set as 20 Hz to accommodate the cloud radar operated
 297 at Atmospheric radiation measurement (ARM) program with approximately 40 spectra being
 298 averaged in 2s (Kollias et al., 2005).

299

300 **3.3 Doppler spectra comparison from two simulators**

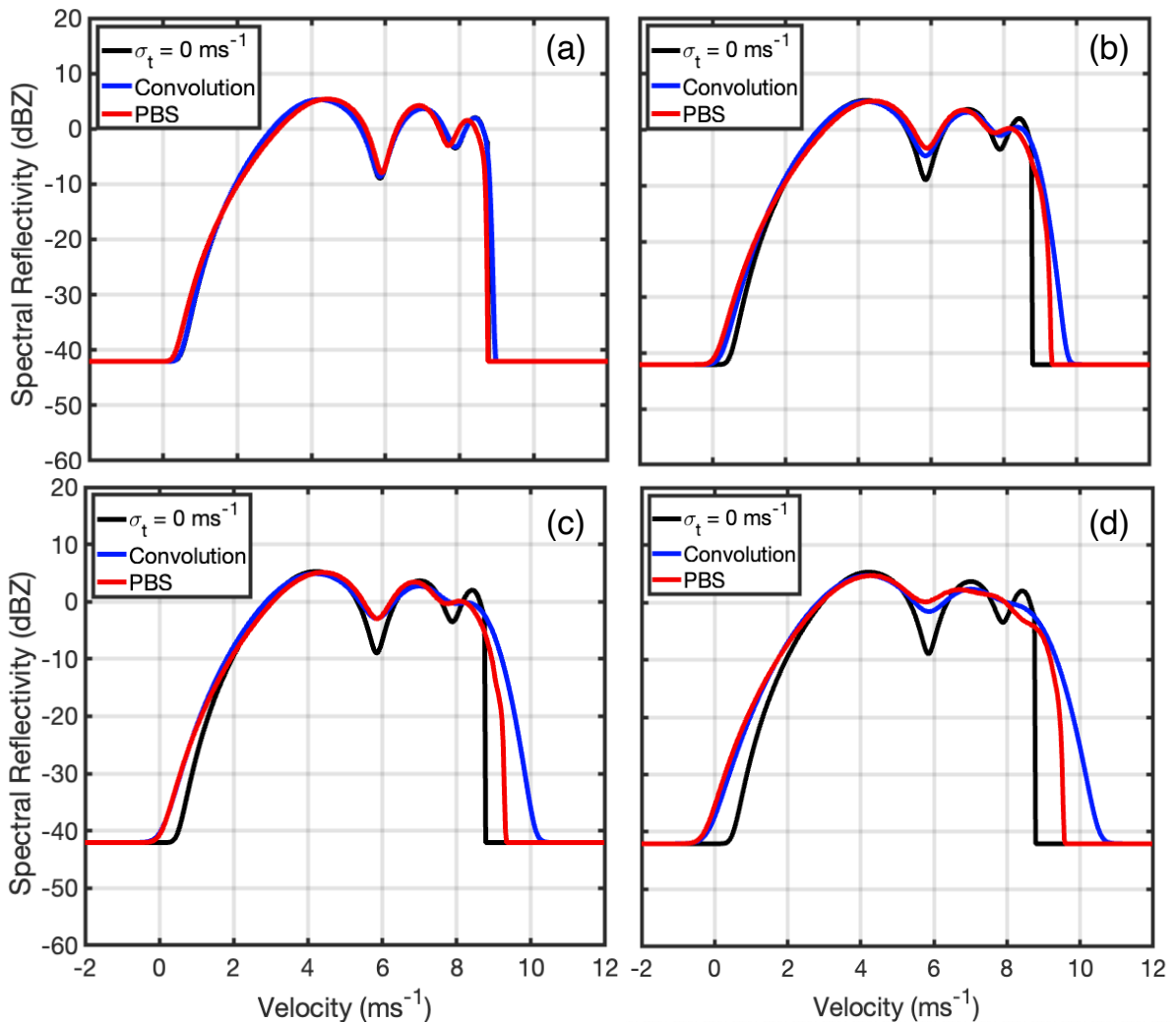
301 Both simulators described above are applied to emulate the Doppler spectrum observed by
 302 a 94-GHz (W-band) profiling cloud radar for a given DSD and for a set of different turbulence
 303 environments. The Nyquist velocity is set as $\pm 12 \text{ ms}^{-1}$ and a 512-point Fast Fourier Transform
 304 (FFT) is used to generate the radar Doppler spectrum. The Marshall-Palmer exponential DSD
 305 (Marshall and Palmer, 1948) with $N(D) = N_0 e^{-\Lambda D}$ is used to represent the DSD in the radar
 306 sampling volume. The values of the intercept parameter N_0 and the slope factor Λ are chosen to be
 307 0.08 cm^{-4} and 15 cm^{-1} . Droplet diameter ranges 10 to 4000 μm with bin size as 1 μm . The
 308 selection of W-band radar and the use of a rain DSD is because it is well known that the W-band
 309 radar Doppler spectrum in rain has distinct features which allow to pinpoint the Doppler spectrum
 310 morphology. Specifically, according to the Mie scattering theory, radar backscattering cross

311 section varies in an oscillatory manner with particle size (Mie, 1908). With the 3.2 mm wavelength
312 radar, the backscattering cross section as a function of droplet size is characterized as several local
313 minimal values with diameter of 1.66, 2.86 mm, which are corresponding to still-air terminal fall
314 velocity of 5.83, 7.89 ms^{-1} . This unique feature is known as “Mie notches” in the radar Doppler
315 spectrum (Kollias et al., 2002;Kollias et al., 2007;Courtier et al., 2022). In the simulation,
316 turbulence field is generated with 20 Hz frequency (f),100s duration (T) and standard deviation
317 (σ) of 0.05 ms^{-1} , 0.25 ms^{-1} , 0.35 ms^{-1} and 0.45 ms^{-1} , respectively.The reason of applying different
318 turbulence settings is to better illustrate the droplet inertia effect under different turbulence
319 environment. It is expected that with increasing turbulence intensity the droplet inertia effect will
320 be manifested in larger differences between the generated radar Doppler spectrum from two
321 methods.

322 When solving the ordinary differential equations described in Section 2.1, the initial droplet
323 velocity is set as 0 ms^{-1} , thus at the beginning of the simulation the droplet gravity force is greater
324 than the wind drag force, the droplet will accelerate until their terminal fall velocity is reached,
325 after which the droplets fluctuate around the terminal fall velocity with variations induced by the
326 exerted wind. The radar Doppler spectrum should be estimated after the steady state is reached.
327 Here, we split the 100s simulated period to two parts: the first 40s is the “speed-up” time which
328 allows the droplets of different size adjust to their steady state, the remaining 60s is used for
329 Doppler spectrum emulation. Specifically,each Doppler spectrum is estimated within a 2s interval
330 as illustrated in Section 3.2, then the generated 30 Doppler spectra in the 60s are further averaged
331 to produce the final Doppler spectrum. This final average step is used to smooth the Doppler
332 spectrum generated in a short period (2s) during which the averaged exerted wind may have a non-
333 zero value.

334 The emulated Doppler spectrum from two methods with four turbulence settings are shown
335 in Figure 5. In a turbulent environment with σ_t of 0.05 ms^{-1} (Figure 5a), the two simulated spectra
336 (red and blue line in Figure 5a) and the Doppler spectrum without turbulence broadening (black
337 line) are almost overlapping with each other, indicating that the radar Doppler spectrum shape is
338 dominated by the DSD shape and the droplets still-air terminal fall velocity in weak turbulence
339 conditions. For σ_t equal to 0.25 ms^{-1} , the broadening of the right edge of the radar Doppler
340 spectrum from the physics-based simulation(PBS) approach (red line in Figure 5b) is less than that
341 produced with the convolution approach (blue line in Figure 5b). As σ_t increases to 0.35 ms^{-1} , a

342 large differences between the right edges of the spectra from the two simulators can be clearly
 343 identified. When σ_t reaches to 0.45 ms^{-1} , the right edge velocity difference between two spectra is
 344 larger than 1 ms^{-1} . Overall, the right edge from the PBS-generated Doppler spectrum is more steep
 345 than that from the convolution-based approach, illustrating that large droplets can not follow the
 346 rapidly changed turbulent field due to the inertia effect. Another notable finding is the left part of
 347 Doppler spectra (velocity smaller than 4 ms^{-1}) from two simulators almost overlap with each other
 348 in different turbulence scenarios, as this part of the spectrum is mostly contributed by small
 349 droplets with negligible inertial effect, thus the corresponding Doppler spectrum can be adequately
 350 represented by the convolution process.



351
 352 Figure 5. Doppler spectrum generated by the convolution-based (blue line) and physics-based
 353 simulation (PBS) (red line) approach for turbulence standard deviation with (a) 0.05 ms^{-1} , (b) 0.25

354 ms^{-1} , (c) 0.35 ms^{-1} , (d) 0.45 ms^{-1} . The black line represents generated Doppler spectrum with $\sigma_t =$
355 0 ms^{-1} . Positive velocity indicates downward motion.

356

357 Comparing the three generated Doppler spectra in Figure 5, we can clearly identify the
358 effect of droplet inertia on Doppler spectrum morphology under different turbulence environments.
359 In general, both simulators indicate a wider Doppler spectrum under a large turbulence condition,
360 but with different broadening magnitudes. The convolution-based approach generates a wider
361 spectra in a more turbulent environment. This overestimation of the turbulence broadening effect
362 indicates that the convolution process used in the conventional simulator is unable to accurately
363 represent the interaction between DSD and turbulence field. On the other hand, for the small
364 droplets, the inertial effect is negligible and the generated Doppler spectra from two approaches
365 are consistent with each other. It is therefore concluded that the convolution process can simulate
366 the Doppler spectrum for the light drizzle precipitation which mostly occurs in marine boundary
367 layer clouds but it is inadequate to emulate Doppler spectrum for the heavy precipitation in deep
368 convection, especially in the presence of strong turbulence environment.

369

370 **4 An illustrative example of Doppler spectrum comparison between observation and** 371 **simulation**

372 In this section, we will present an illustrative example by using one observed Doppler
373 spectrum to evaluate the performance of the simulators. The observed Doppler spectrum is
374 obtained from the W-band ARM Cloud Radar (WACR) at the ARM Southern Great Plain (SGP)
375 observatory during a heavy precipitation period on May 9, 2007. For the WACR, the maximum
376 unambiguous velocity is 7.8 ms^{-1} , which is smaller than the still-air terminal velocity of droplets
377 with diameter larger than 3 mm and lead to velocity folding. Here velocity de-aliasing process is
378 performed to reconstruct the Doppler spectrum with velocity from 0 ms^{-1} to 11 ms^{-1} . The observed
379 Doppler spectrum is further calibrated from the displacement caused by vertical air motion by
380 pinpointing the location of first Mie notch of the Doppler spectrum to 5.83 ms^{-1} . (Kollias et al.,
381 2002). To simulate the Doppler spectrum, the hydrometeor DSD and the turbulence broadening
382 term (σ_t) are needed. Here, the raindrops DSD is observed from the impact disdrometer which can
383 measure droplet diameter from 0.3 mm to 5.4 mm with 20 bins (Wang et al., 2021). The temporal
384 resolution of the WACR and the disdrometer is 4.28s, 1min respectively. To make the observation

385 from two instruments comparable, the WACR-observed Doppler spectra are averaged over 1min
 386 to coincide with the disdrometer observational period. For this example, we use the disdrometer-
 387 measured DSD from 05:44 to 05:45 UTC to simulate the radar Doppler spectrum and compare it
 388 with the one observed of WACR in the same period.

389 The observed DSD is shown in Figure 6a, and the corresponding WACR-observed Doppler
 390 spectrum is shown as the black line in Figure 6b. Based on the observed DSD, the radar Doppler
 391 spectrum for the droplets falling in still air is generated (not shown), from which the DSD-
 392 contributed Doppler spectrum width (σ_D) is estimated as 1.34 ms^{-1} . Since the wind shear
 393 broadening contribution (σ_S) to radar Doppler spectrum is generally smaller than σ_D and the
 394 turbulence broadening (σ_t) (Borque et al., 2016), here we neglect the σ_S contribution and estimate
 395 σ_t as:

$$396 \quad \sigma_t^2 = \sigma_O^2 - \sigma_D^2$$

397 Where σ_O is the observed Doppler spectrum width, which is 1.46 ms^{-1} in this example, and
 398 σ_t is estimated as 0.58 ms^{-1} . To estimate the accuracy of σ_t , we further assume the observed DSD
 399 is the only source of the uncertainty. Considering that the accuracy of the droplets size
 400 measurement of the disdrometer is approximately $\pm 5\%$ (Wang et al., 2021), the uncertainty of σ_D
 401 and σ_t is estimated as 0.15 ms^{-1} .

402 With the observed DSD and the estimated σ_t , the radar Doppler spectrum can be simulated.
 403 It is noted that large rain droplets falling in the air are nonspherical, thus backscattered power from
 404 an oblate droplet may be different from the one from rigid liquid sphere. To this end, for the Mie
 405 scattering calculation, axis ratio ($\frac{a}{b}$) of the droplet with diameter larger than 2mm is considered
 406 as a function of diameter (D) with unit of mm (Pruppacher and Beard, 1970):

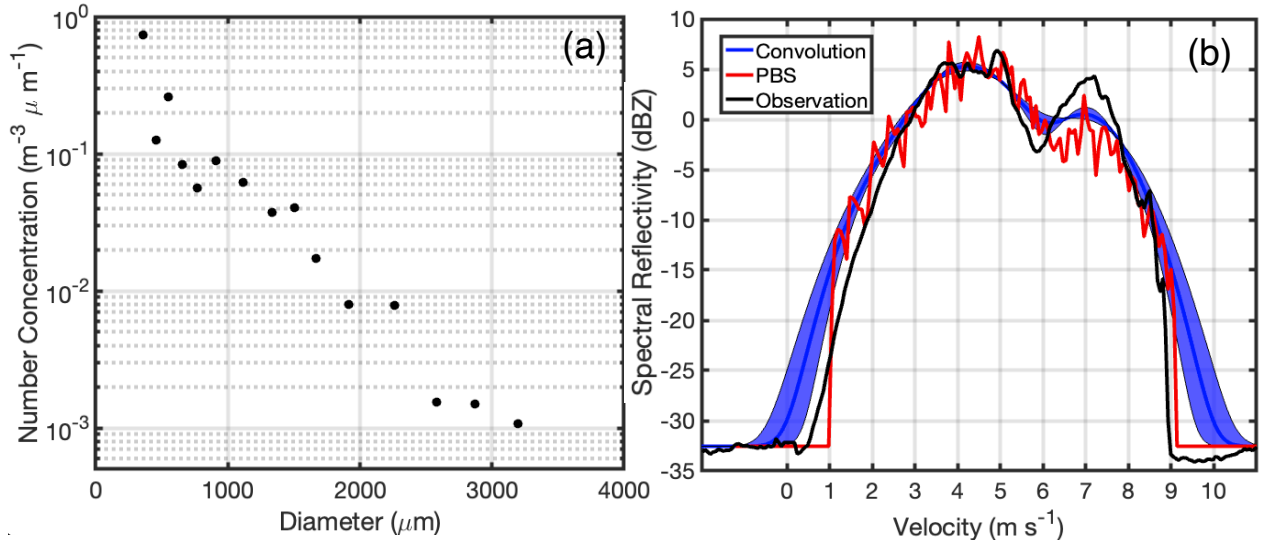
$$407 \quad \frac{a}{b} = 1.03 - 0.062D$$

408 The simulated Doppler spectrum from the convolution and the PBS method are shown in
 409 Figure 6(b). It is noticeable that the Doppler spectrum from the PBS approach (red line) is more
 410 noisy than that from the convolution approach (blue line). This is due to the insufficient bin
 411 categories of the particle measured from disdrometer, it is expected that with increasing the number
 412 of measured particle size, the generated Doppler spectrum become more smooth. Nevertheless, it
 413 is still recognizable that the both the morphology and the magnitude of the PBS-based spectrum
 414 right edge is more consistent with observation compared with the one generated from the

415 convolution approach. Both of the two simulators represent the first peak of the Doppler spectrum
416 from 3 ms^{-1} to 6 ms^{-1} very well, while neither of them generate a consistent second peak
417 morphology compared with observation. The left edge of the Doppler spectrum from the
418 convolution-based approach is broader than the observation, while the PBS is unable to represent
419 the Doppler spectrum smaller than 1 ms^{-1} due to the absent of the droplets with diameter smaller
420 than 0.3 mm observed from disdrometer.

421 The purpose of this Doppler spectrum comparison is not for a robust validation but used as
422 an illustrative example to show the morphology of the simulated Doppler spectrum based on real
423 observations and to discuss the required measurements would be used for robust Doppler spectrum
424 simulator validation. To a certain degree, a more consistency Doppler spectrum morphology is
425 identified between the observation and from the PBS simulator, especially for the right edge of the
426 spectrum. However, great cautions should be taken for further interpretation as both of the
427 simulators cannot represent the left part of the Doppler spectrum and the second notches very well.
428 This discrepancy is mainly because the observed DSD by disdrometer may not an adequate
429 representation of the hydrometeors that contribute the Doppler spectrum observed by WACR.
430 Specifically, there are three critical challenging issues should be overcome before a solid and
431 convincing Doppler spectrum simulator evaluation effort being performed: 1) the disdrometer is
432 located at the surface, while the lowest measurement height of WACR is 460m. When the rain
433 droplets fall, droplets may collide, breakup, and being advected from adjacent regions by the
434 horizontal wind; Thus a large uncertainty is expected by using the surface-observed DSD to
435 represent the hydrometeor distribution at 450m above; 2) the observed DSD from the disdrometer
436 only measure droplets with 20 size categories, which is insufficient for the physics-based
437 simulation to generate a smooth and complete Doppler spectrum; 3) the uncertainty of the
438 estimated σ_t is challenging to be well constrained due to the large uncertainty of the observed DSD
439 mentioned above. A comprehensive and solid validation of the Doppler spectrum simulator require
440 simultaneous and well- aligned DSD and Doppler spectrum measurement; large number of the
441 measured droplet size categories and carefully estimation of the environment turbulence
442 broadening factors.

443



444 Figure 6. (a) Black dots represent the observed raindrop number concentration from disdrometer
 445 at 05:44 (UTC) on May 9, 2007 on SGP site. (b) Doppler spectra simulated from the PBS (red)
 446 and convolution (blue) method and the observed spectrum from WACR (black line). The blue
 447 shaded region represents the uncertainty of the simulated Doppler spectrum produced by the
 448 uncertainty in σ_t based on the convolution method. Positive velocity indicates downward motion.
 449

450 5 Conclusions

451 The radar Doppler spectrum offer unprecedented capabilities for studying cloud and
 452 precipitation microphysics. Recent advancements in radar technology and signal processing have
 453 enable the continuous recording of high-quality radar Doppler spectra observations from a wide
 454 range of profiling radar systems (Kollias et al., 2005; Kollias et al., 2016). Until now, the simulation
 455 of the radar Doppler spectra was based on well-established techniques (Gossard, 1988; Kollias et
 456 al., 2011a). However, inertial effect of large droplets are typically neglected in the design of current
 457 simulators. Here, the impact of the liquid droplet's inertia on the shape of the radar Doppler
 458 spectrum was investigated. A physics-based simulation framework is developed to simulated the
 459 droplets velocity in a given turbulence environment. It demonstrates that big droplets with large
 460 inertia will take longer time to adapt to the change of velocity field, indicating large droplets are
 461 incapable of following the turbulence wind as small droplets do.

462

463 Building on the simulation framework, a new approach is proposed to emulate Doppler
 464 spectrum by simulating the velocity of each droplet during the entire time domain. The simulated

465 W-band radar Doppler spectrum is compared with the one generated from the traditional method
466 for a typical DSD with four different turbulence environments. The comparison indicates that the
467 traditional Doppler simulator without considering the inertial effect generates an artificially
468 broader Doppler spectrum. This inertia effect becomes more noticeable as turbulence intensity
469 increases. This finding suggests that special caution should be taken when applying convolution-
470 based approaches to represent DSD-turbulence interaction in heavy precipitation. In the case of
471 light precipitation mostly happening in marine boundary layer cloud, the droplet inertia effect on
472 Doppler spectrum is negligible and the traditional simulator generates consistent results with the
473 proposed simulator.

474 One WACR-observed Doppler spectrum collected from the ARM SGP observatory is
475 compared with the simulated Doppler spectrum as an illustrative example to validate the fidelity
476 of the simulator from the convolution and the PBS-based approach. The presented case shows that
477 the proposed PBS generate a more similar morphology of the right edge of the Doppler spectrum
478 compared with the traditional simulator. However, both of two simulator fail to reconstruct the left
479 edge and the second notch of the Doppler spectrum. These inconsistencies are due the fact that the
480 surface-based DSD from disdrometer is inadequate to represent the hydrometeor observed by
481 cloud radar at a high level. A careful and solid validation of the radar Doppler spectrum simulator
482 would require co-aligned observations of DSD and Doppler spectrum and well-constrained
483 turbulent broadening estimations. Nevertheless, the proposed Doppler spectrum simulator, with
484 the ability to simulate individual droplet motion as well as their manifestation on Doppler spectrum,
485 provide an valuable tool to improve the understanding of Doppler radar observation from a
486 fundamental physics perspective. We expect this proposed Doppler spectrum simulation
487 framework can stimulate more studies to better interpret the Doppler radar observation and to
488 decode the microphysics and dynamics information concealed in radar Doppler spectrum.

489

490 **Competing interests.**

491 **P. K.** is the associate editor of AMT and the peer-review process was handled by an independent
492 editor. The authors have no other competing interests to declare.

493

494

495 **Code/Data availability**

496 The codes of the proposed Doppler spectrum simulator can be accessed via

497 <https://doi.org/10.5281/zenodo.7897981>.

498 Ground-based data were obtained from the Atmospheric radiation measurement (ARM) user
499 facility, a U.S. Department of Energy (DOE) Office of Science user facility managed by the Office
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508

509

510 **Author contributions**

511 Zeen Zhu implemented the method, performed the analysis, produced the figures, and wrote the
512 initial draft of the manuscript. Pavlos Kollias supervised and provided advice and guidance on all
513 aspects of the analysis and contributed to the writing of the manuscript. Fan Yang advised on
514 results interpretation and manuscript editing. All authors read the manuscript draft and contributed
515 comments.

516

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