

# Benchmarking data-driven inversion methods for the estimation of local CO<sub>2</sub> emissions from XCO<sub>2</sub> and NO<sub>2</sub> synthetic satellite images

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## Abstract.

The largest anthropogenic emissions of carbon dioxide (CO<sub>2</sub>) come from local sources such as cities and power plants. The upcoming Copernicus CO<sub>2</sub> Monitoring Mission (CO2M) will provide satellite images of the CO<sub>2</sub> and NO<sub>2</sub> plumes associated with these sources at a resolution of 2 km × 2 km and with a swath of 250 km. These images could be exploited with atmospheric plume inversion methods to estimate local CO<sub>2</sub> emissions at the time of the satellite overpass and the corresponding uncertainties. To support the development of the operational processing of satellite column-averaged CO<sub>2</sub> dry air mole fraction (XCO<sub>2</sub>) and tropospheric column NO<sub>2</sub> imagery, this study evaluates “data-driven inversion methods”, i.e., computationally light inversion methods that directly process information from satellite images, local winds and meteorological data, without resorting to computationally expensive dynamical atmospheric transport models. We have designed an objective benchmarking exercise to analyse and compare the performance of five different data-driven inversion methods: two implementations with different complexity for the cross-sectional flux approach (CSF and LCSF) and one implementation for the Integrated Mass Enhancement (IME), the Divergence (Div) and the Gaussian Plume model inversion (GP) approaches. This exercise is based on pseudo-data experiments with simulations of synthetic “true” emissions, meteorological and concentration fields, and CO2M observations in a domain of 750 km × 650 km centred on Eastern Germany over 1-year. The performance of the methods is quantified in terms of accuracy in the single-image (from individual images) or annual average (from the full series of images) emission estimates and in terms of number of instant estimates for the city of Berlin and 15 power plants in this domain. Several ensembles of estimations are conducted, using different scenarios for the available synthetic datasets. These ensembles are used to analyse the sensitivity of the performance to the loss of data due to cloud cover, to the uncertainty in the wind or to the added value of simultaneous NO<sub>2</sub> images. The GP and the LCSF methods generate the most accurate estimates from individual images. The deviations between the emission estimates and the true emissions from these two methods have similar Interquartile Ranges (IQR):

32 between ~20% and ~60% depending on the scenario. When taking the cloud cover into account, these methods produce  
33 respectively 274 and 318 instant estimates from the ~500 daily images that cover significant portions of the plumes from the  
34 sources. Filtering the results based on the associated uncertainty estimates can improve the statistics of the IME and CSF  
35 methods, but at the cost of a large decrease in the number of estimates. Due to a reliable estimation of uncertainty and thus a  
36 suitable selection of estimates, the CSF method achieves similar if not better statistics of accuracy for instant estimates  
37 compared to the GP and LCSF methods after filtering. In general, the performances for retrieving single-image estimates are  
38 improved when, in addition to XCO<sub>2</sub> data, collocated NO<sub>2</sub> data are used to characterise the structure of plumes. With respect  
39 to the estimates of annual emissions, the root mean square errors (RMSE) are for the most realistic benchmarking scenario  
40 20% (GP), 27% (CSF), 31% (LCSF), 55% (IME) and 79% (Div). This study suggests that the Gaussian plume and/or the  
41 cross-sectional approaches are currently the most efficient tools to provide estimates of CO<sub>2</sub> emissions from satellite images  
42 and their relatively light computational cost will enable analysis of the massive amount of data provided by future missions  
43 of satellite XCO<sub>2</sub> imagery.

## 44 **1 Introduction**

45 The satellite imagery of column-averaged CO<sub>2</sub> dry air mole fractions (XCO<sub>2</sub>) has been identified as an essential  
46 component of a future atmospheric observing system to monitor anthropogenic CO<sub>2</sub> emissions, and in particular to detect  
47 and monitor hotspot atmospheric plumes and thus emissions, in order to verify emission reductions or assess national  
48 budgets (Ciais et al., 2015; Pinty et al., 2017). The Copernicus CO<sub>2</sub> Monitoring (CO2M mission was designed to meet these  
49 objectives with a constellation of two to three Low Earth Orbit (LEO) satellites flying in a sun-synchronous low-earth orbit  
50 crossing the Equator around 11:30 local time. Each satellite will carry an imaging spectrometer providing images of XCO<sub>2</sub>  
51 and of NO<sub>2</sub> tropospheric column densities (referred to as NO<sub>2</sub> hereinafter) along a 250 km wide swath with a resolution of 2  
52 km × 2 km (Sierk et al., 2019). Current satellite missions, like Sentinel-5 Precursor (Sentinel-5P) and the third Orbiting  
53 Carbon Observatory (OCO-3, when targeting specific sources in its Snapshot Area Map -SAM- mode), already deliver NO<sub>2</sub>  
54 column-density and XCO<sub>2</sub> images, albeit, for the former, at a resolution coarser than CO2M, and for the latter, over areas  
55 and at a frequency much smaller than with CO2M. Upcoming missions, such as Global Observing SATellite for Greenhouse  
56 gases and Water cycle (GOSAT-GW, Kasahara et al., 2020), MicroCarb (in its “city-mode”, Pascal et al., 2017) and Twin  
57 ANthropogenic Greenhouse gas Observers (TANGO, Landgraf et al., 2020), are expected to increase the amount of CO<sub>2</sub> and  
58 NO<sub>2</sub> images of the plumes from emission hotspots.

59 Operational services are being developed such as the Copernicus capacity for anthropogenic CO<sub>2</sub> emissions monitoring  
60 and verification support (CO2MVS, Pinty et al., 2017; Janssens-Maenhout et al., 2020), to process these XCO<sub>2</sub> and NO<sub>2</sub>  
61 images for the monitoring of emissions in a systematic and global way at spatial and time scales that are relevant for  
62 policymakers and to support emission mitigation actions. Plume inversion systems are used to derive estimates of the CO<sub>2</sub>  
63 emissions from local sources using satellite images of the corresponding atmospheric plumes. One of the key elements of

64 operational services will thus be standard plume inversion methods providing precise and reliable data in an automated and  
65 fast manner. Various plume inversion approaches and implementations are now regularly used to process the existing  
66 spaceborne atmospheric plumes images (Varon et al., 2018; Zheng et al. 2020; Kuhlmann et al., 2021; Nassar et al., 2021;  
67 Jacob et al., 2022; Hakkarainen et al., 2023a). Therefore, there is a need to benchmark in a quantitative way the plume  
68 inversion methods for the estimation of local emissions of CO<sub>2</sub>, and more generally of greenhouse gases and pollutants.

69 Monitoring anthropogenic CO<sub>2</sub> emissions of point sources or cities from satellite XCO<sub>2</sub> images is challenging as  
70 corresponding column-average enhancements are often small compared with the local fluctuations of the “background” CO<sub>2</sub>  
71 field due to biogenic CO<sub>2</sub> fluxes and to neighbour anthropogenic sources, and with the typical level of errors in the XCO<sub>2</sub>  
72 retrievals (Buchwitz et al., 2013). Despite this challenge, the potential of CO<sub>2</sub> imagers to estimate anthropogenic emissions  
73 has been demonstrated with observing system simulation experiments (OSSEs) using synthetic data, for power plants  
74 (Bovensmann et al., 2010), cities (Pillai et al., 2016; Broquet et al., 2018; Wang et al., 2020) and in a more general way, at  
75 local to national scales (Santaren et al., 2021). Furthermore, several studies have shown that the joint analysis of co-located  
76 NO<sub>2</sub> satellite observations strongly enhances the skill to detect the XCO<sub>2</sub> enhancement plumes from sources in XCO<sub>2</sub>  
77 images, and consequently to estimates the corresponding CO<sub>2</sub> emissions (Reuter et al., 2019; Kuhlmann et al., 2021). NO<sub>2</sub>  
78 observations are indeed characterised by a better signal-to-noise ratio and a generally small and low-amplitude background  
79 field, due to the relatively short lifetime of nitrogen oxides (NO<sub>x</sub>).

80 CO<sub>2</sub> emissions of large point sources and cities can be estimated from satellite images by plume inversion systems  
81 integrating the observations with dynamical transport model simulations of atmospheric CO<sub>2</sub> concentrations (e.g., Broquet et  
82 al., 2018; Ye et al., 2020; Santaren et al., 2021). In principle, the use of such dynamical models could support the analysis of  
83 the 3D dynamical patterns of the observed plume and thus the accuracy of the inversion. They could also support the  
84 derivation of the spatial distribution of the emissions within cities, and of the temporal variation of the emissions  
85 corresponding to a plume in the hours preceding each satellite overpass. However they can be strongly impacted by  
86 modelling errors which become critical at local scale, when trying to model plumes from emission hotspots over a few tens  
87 to a few hundreds of kilometres (Brunner et al., 2023). Furthermore, their computational burden hampers their use for a  
88 global and routine coverage of the sources in an operational context. *Data-driven plume inversion methods* appear to be  
89 currently more suitable for such wide-scale applications (Ehret et al., 2022). These are computationally light inversion  
90 methods that directly process information from satellite images and local winds and meteorological data (typically from  
91 operational weather analyses), without resorting to dynamical atmospheric transport models.

92 The main data-driven approaches for estimating local emissions based on satellite images of plumes that have been tested  
93 and analysed in a significant number of studies are:

94 1) the Integrated Mass Enhancement (IME) approach, which relates the total mass of plumes to the corresponding  
95 emissions; it has been used for retrieving CH<sub>4</sub> emissions from airborne observations (Frankenberg et al., 2016) or from fine-  
96 scale satellite data (Varon et al., 2018)

97 2) the Gaussian plume approach which extracts emissions from the fit of plume shapes by Gaussian functions and was  
98 applied for instance to estimate power plant CO<sub>2</sub> emissions from OCO-2 satellite data (Nassar et al. 2017; 2021)

99 3) the cross-sectional flux approach which infers emissions from the fluxes passing through cross-sections of the plumes  
100 and whose potential to estimate CO<sub>2</sub> emissions of power plants with CO<sub>2</sub> and NO<sub>2</sub> satellite imagery data was assessed, for  
101 instance, by Kuhlmann et al. (2021)

102 4) the divergence (Div) approach, which derives emissions from the application of the divergence operator to fields of  
103 fluxes and which was originally designed to estimate nitrogen oxide (NO<sub>x</sub>) emissions from NO<sub>2</sub> data provided by the  
104 TROPOMI satellite imagery (e.g. Beirle et al., 2019; 2021, 2023) and was more recently adapted to the quantification of CO<sub>2</sub>  
105 emissions (Hakkarainen et al., 2022). Contrarily to the other methods of this study, the Div method is generally used to  
106 generate annual estimates from average fields extracted from multiple images.

107 Against this background, the aim of this study is to benchmark these four data driven plume inversion approaches for the  
108 monitoring of CO<sub>2</sub> emission hotspots with CO2M images. We present a benchmarking framework to objectively evaluate  
109 and compare the performance of different implementations of the four data-driven approaches (Sect. 2.1) to estimate CO<sub>2</sub>  
110 local emissions from such satellite data. For this purpose, we use one year of synthetic satellite observations closely  
111 mimicking those expected from the upcoming CO2M mission (Sect. 2.2) that were generated in the European Space Agency  
112 (ESA) funded SMARTCARB project from high-resolution atmospheric transport simulations (e.g. Brunner et al., 2019;  
113 Kuhlmann et al., 2020). The emissions of the city of Berlin and 15 large power plants are estimated from these synthetic  
114 satellite data and the ability of the different inversion methods is assessed by comparing their estimates to the corresponding  
115 *true* values used by the atmospheric transport model. Performances of the different inversion approaches are evaluated for 1)  
116 single-image estimates that are retrieved from daily images (Sect. 3) and, 2) annual estimates that are computed from the  
117 inversion of one year of data (Sect. 4). Furthermore, performances are analysed for different scenarios regarding the data  
118 used by the inversions, where the impacts of considering the cloud cover in the data, the uncertainties in the wind and the use  
119 of collocated NO<sub>2</sub> data are assessed. Finally, results are discussed by analysing 1) the potential of ensemble approaches that  
120 would gather different inversion methods and, 2) the trade-off between overall accuracy and number of estimates when the  
121 cases are filtered based on the uncertainties in the estimates computed by the plume inversion methods (Sect. 5).

## 122 **2 Data and methods**

### 123 **2.1 Data-driven inversion methods**

124 Five different emission quantification methods are evaluated in this study: (1) the integrated mass enhancement method  
125 (IME), (2) the cross-sectional flux (CSF) method, (3) the light cross-sectional flux (LCSF) method, (4) the Gaussian plume  
126 (GP) method and (5) the divergence (Div) method. More precisely, what is studied here are specific configurations of certain  
127 methods as is the case for the CSF and LCSF “methods” which are derived from the same general approach. But, hereinafter  
128 we will refer to these configurations as methods to avoid weighing down the text. The general approaches have been widely

129 used and described in previous papers such as Varon et al. (2018) and Beirle et al. (2019, 2021). The specific  
130 implementations of the CSF and Div methods tested here have been used extensively by the authors in previous studies  
131 (Kuhlmann et al., 2019, 2020, 2021 and Hakkarainen et al., 2022). They have been slightly upgraded in the course of this  
132 benchmarking exercise to improve their stability, accuracy, and capability of running in a fully automated way. Details of the  
133 methods are presented in an accompanying study by Kuhlmann et al. (2023). Further details about the theory of the Div  
134 method and its application are given in Koene et al. (2023) and Hakkarainen et al. (2022, 2023b). All algorithms and tools  
135 used in this work have been integrated into a Python library for *data-driven emission quantification* (ddeg), which has been  
136 made publicly available and is described in Kuhlmann et al. (2024). We provide below a short description of these methods  
137 with an emphasis on their relative advantages and limitations and on the way they estimate uncertainty. The main features of  
138 the methods are summarised in Table 1 and illustrated in Figure 1 and Figure A1. Table 1 also lists the computation times of  
139 the methods calculated for the same inversion example using the same hardware. As the methods have all been implemented  
140 in the same Python package, the timings are directly comparable.

141 All methods except the Div method can provide estimates derived from individual satellite images. The Div approach as  
142 implemented here is based on the averaging of information contained within multiple images and hence typically delivers  
143 annual estimates. We will hereinafter refer to the IME, CSF, LCSF and GP methods as single-image methods. These  
144 methods share a common algorithmic sequence that starts with identifying clusters of enhancements above a background in  
145 satellite images. Subsequently, these clusters are assigned to plumes from specific known sources, and finally, the emissions  
146 of the corresponding sources are estimated. The plume detection combines the first two stages and can be used to discern  
147 plumes from unreported sources; however the ability of the different approaches to detect unknown point sources has not  
148 been studied here, as the primary focus is to analyse their potential to detect and process plumes of known sources from  
149 CO2M-like satellite images (see Sect. 2.2). Of mention is that the divergence, cross-sectional flux and machine-learning  
150 approaches are particularly well-suited for automatic detection of plumes from unknown sources (Zheng et al., 2020; Beirle  
151 et al., 2021; Schuit et al., 2023). Moreover, as previously mentioned, a benefit of the CO2M mission is the availability of co-  
152 registered XCO<sub>2</sub> and NO<sub>2</sub> columns, which can further benefit the plume detection and emission quantification steps.

153 Obtaining the column enhancements over the background can be achieved with different thresholding techniques as  
154 detailed below. When it comes to NO<sub>2</sub>, the global background field is insignificant but in the case of CO<sub>2</sub>, its amplitude is  
155 important and can vary significantly in space and time due to biogenic and other anthropogenic fluxes surrounding the  
156 sources of interest and due to gradients in the background. Another common feature is the need for defining an effective  
157 wind speed, which describes the average mass transport of CO<sub>2</sub> within the plumes. This a major challenge as wind speed  
158 varies with altitude whereas satellite images contain integrated column measurements with no vertical resolution.  
159 Additionally, the horizontal resolutions of wind products are generally different from those of satellite images. To address  
160 these limitations, the methods determine effective winds in a more or less sophisticated manner.

161 Finally, all methods have implemented some quality control on their estimates. These checks are more or less restrictive  
162 depending on the methods and may filter out, for example, cases with overlapping plumes originating from neighbouring

163 sources. Further details are provided in Kuhlmann et al. (2023). Of particular note is the fact that our implementation of the  
164 GP method discards values that are below 1/4 or beyond 4 times the “true” values averaged one hour before the satellite  
165 overpass (10:00 to 11:00 UTC); this filtering stabilises the otherwise underdetermined inversion. Unlike the other methods,  
166 the GP method thus uses a priori information about the source strength, which artificially improves its performance.

### 167 **2.1.1 Cross-sectional flux (CSF) inversion method**

168 The cross-sectional flux inversion method has been used in many studies such as for example the determination of CH<sub>4</sub>  
169 emissions of point sources from high-resolved satellite data for which its superiority over other methods has been  
170 demonstrated within the framework of the study of Varon et al. (2018). In brief, this method calculates the fluxes through  
171 single or multiple cross-sections of the plumes as the product of effective winds and integrals of column mass enhancements  
172 along plume transects (line densities). Under the assumption of steady-state conditions, these fluxes are equivalent to the  
173 emissions. The CSF method used in this study has been used by Kuhlmann et al. (2020, 2021) for the estimation of CO<sub>2</sub>  
174 emissions from CO<sub>2</sub> and NO<sub>2</sub> images. These studies have demonstrated that the inclusion of NO<sub>2</sub> observations significantly  
175 increases the number and precision of the estimates.

176 The plume detection module of the CSF approach determines in a first stage the CO<sub>2</sub> or NO<sub>2</sub> pixels that are significantly  
177 enhanced above the background with a statistical z-test (Kuhlmann et al., 2021). To perform this, a Gaussian kernel to  
178 average local observations values is applied and the background field is at this stage computed by applying a median filter.  
179 The parameters defining the z-test were carefully assessed in order to get enough valid pixels to describe a plume while  
180 avoiding false detections (Kuhlmann et al. 2019). The detected pixels are then grouped by a labelling algorithm and assigned  
181 to a source. Finally, a curve representing the centerlines of the plume is fitted to the detected pixels.

182 For the quantification of CO<sub>2</sub> emissions, the CSF method groups the detected plume pixels into sub-polygons along the  
183 curved plume, whose width equals ~5 km (2-3 pixels of CO<sub>2</sub>M data). All detected pixels within a sub-polygon are used to  
184 construct a single estimate of the line density. Following Reuter et al. (2019), the CSF method assumes that the plume  
185 transect follows a Gaussian behaviour, after removing the background signal with a normalised convolution. To obtain the  
186 line densities, the integration of the fitted Gaussian functions does not require any additional computation as the line  
187 integrals are simply equal to the amplitude parameters of the fitted Gaussian functions. Then, in order to be converted into  
188 fluxes, line densities are multiplied by effective winds which are the horizontal winds at the corresponding source locations  
189 and times of the satellite overpasses, vertically weighted by the GNFR-A/SNAP-1 emission profile (Brunner et al., 2019).

190 Finally, the CO<sub>2</sub> emission of a given source retrieved from a given satellite image is computed by averaging the CO<sub>2</sub>  
191 estimated fluxes of all the sub-polygons describing the plume downstream of the source. The uncertainty in the emission  
192 estimate is then computed by propagation of the uncertainties in the line densities computation and in the wind; the  
193 uncertainties in the line densities are extracted from the standard deviation of the sub-polygon estimates and capture mostly  
194 satellite data noise through uncertainty in the Gaussian fitting.

195 When NO<sub>2</sub> data are used in conjunction with CO<sub>2</sub>, detections of plumes are first performed for NO<sub>2</sub>, while the CO<sub>2</sub> and  
196 NO<sub>2</sub> enhancements are fitted simultaneously by Gaussian functions that share the same mean (or central location) and the  
197 same standard deviation. Thus, the fit of CO<sub>2</sub> enhancements takes advantage of the better signal-to-noise ratio of NO<sub>2</sub> data  
198 by better constraining the parameters of the Gaussian functions, which provides more accurate estimates of CO<sub>2</sub> line  
199 densities and hence CO<sub>2</sub> emissions.

### 200 **2.1.2 Light cross-sectional flux (LCSF) inversion method**

201 The light cross-sectional flux method shares the same theoretical foundations as the CSF method, but its implementation  
202 is largely different. It is derived from the method originally developed by Zheng et al. (2020) to estimate the CO<sub>2</sub> emissions  
203 of cities and industrial areas in China that produce atmospheric plumes clearly detectable in transects of OCO-2 data which  
204 are characterised by a resolution of few km<sup>2</sup> and by a swath about 10 km wide, which is almost 25 times narrower than the  
205 ~250 km wide swath of the CO2M instruments. This method has been applied to the routine and automatic estimation of  
206 isolated clusters of CO<sub>2</sub> emissions worldwide (Chevallier et al., 2020) and to study the temporal variability of the emissions  
207 based on several years of OCO-2 and OCO-3 data (Chevallier et al., 2022). The method has undergone significant  
208 modifications for this comparative study, where the location of the emission sources is known, in order to fully harness the  
209 potential of high-resolution satellite imagery.

210 For a given source and satellite overpass, the LCSF method performs a simple detection of the plume by extracting from  
211 the satellite image an area which is 100 km wide in across-wind (perpendicular) direction and which extends downwind the  
212 source over a distance equal to the distance travelled by the wind in one hour. The method then selects the pixels of the  
213 extracted area where XCO<sub>2</sub> or NO<sub>2</sub> enhancements – simply defined as the difference between data values and the average  
214 data of the area – are greater than the spatial variability, i.e. the standard deviation of the data contained within the area.

215 The quantification of the source emission is then performed on each selected enhancement by extracting again a 100 km  
216 wide across-wind area centred at the enhancements and extending 10 km (~5 CO2M pixels) downwind from the  
217 enhancements. The sums of a linear term accounting for large scale variations in the background fields and a Gaussian  
218 function describing the plume cross-section perpendicular to the wind direction are then fitted to the data contained within  
219 these areas. The plume detection and fitting of the enhancements can be carried out in the same way when NO<sub>2</sub> data are  
220 available. And, standard deviations and means of the Gaussian functions fitted with NO<sub>2</sub> data are then used for fitting CO<sub>2</sub>  
221 enhancements; CO<sub>2</sub> data constrain in this case only the amplitudes of the CO<sub>2</sub> Gaussian functions. This allows transferring  
222 information derived from NO<sub>2</sub> data when estimating CO<sub>2</sub> emissions from CO<sub>2</sub> data.

223 CO<sub>2</sub> line densities are, as for the CSF method, derived from the Gaussian functions fitted with CO<sub>2</sub> data and converted  
224 into emission estimates by the multiplication of an effective wind. For the LCSF method, this effective wind is extracted at  
225 the location of the enhancements and at an altitude above ground of 100 m, as preliminary tests have shown that extracting  
226 winds at the altitude of 100 m yields, for the LCSF approach, better inversion results compared to other altitudes or  
227 alternative methods of computing the effective winds. This result may be reflecting a trade-off between the need to account

228 for emission injection heights higher than 100 m when considering isolated power plants, and lower than 100 m when  
229 considering the mix of sources within cities, whose emissions are not dominated by large power plants (Brunner et al.,  
230 2024). The automatic process of sources limits the ability to derive a case by case selection of the height for the wind  
231 extraction, but a finer option for future analysis might be to discriminate this selection as a function of the type of target  
232 (considering at least isolated power plants vs. urban areas).

233 Finally, under steady-state atmospheric conditions, the cross-sectional CO<sub>2</sub> flux derived at each selected enhancement is  
234 equivalent to the upwind source emissions. Therefore, as several enhancements belonging to a same atmospheric signature of  
235 a source are generally processed, the algorithm produces multiple individual estimates of the source emission; the estimate  
236 computed by the method for a given source and from a given image is then computed as the median value of these individual  
237 estimates; the use of the median helping to reduce the impact of outliers. Moreover, uncertainties in the individual estimates  
238 provided by the LCSF method are computed by propagation of the errors derived by the fitting algorithm when generating  
239 the line densities; uncertainties in the final estimates are finally the median of these uncertainties.

### 240 **2.1.3 Gaussian plume (GP) inversion method**

241 The Gaussian plume inversion approach assumes that observed plumes can be described with Gaussian plume models. This  
242 approach has been widely used such as for example in the determination of CH<sub>4</sub> point source emissions (Varon et al., 2018),  
243 the use of OCO-2 data to quantify CO<sub>2</sub> emissions from power plants (Nassar et al., 2017), or in a framework to estimate at  
244 the global scale CO<sub>2</sub> emissions from large cities and point sources (Wang et al., 2020). Compared to previous Gaussian  
245 plume inversions, the GP inversion method used in this work allows the Gaussian plume model (like the CSF method) to  
246 handle curved plumes (see Sect 3.2.1 in Hakkarainen et al., 2023b).

247 The detection of plumes, i.e. of the CO<sub>2</sub> or NO<sub>2</sub> enhancements from the background, is carried out using the same  
248 algorithm as for the CSF method. Then, the inversion uses a Levenberg-Marquardt least-squares optimization to find the  
249 optimal parameters of the Gaussian functions fitting the enhancements and, of the Bézier curves describing the centre lines  
250 of the plumes (Hakkarainen et al., 2023b). If NO<sub>2</sub> data and CO<sub>2</sub> data are simultaneously available, then the Gaussian plume  
251 model is first fitted to the NO<sub>2</sub> observations and the optimised parameters regarding the plume shape are subsequently used  
252 as first guesses for the fitting to CO<sub>2</sub> observations. These derived parameters are constrained to remain close to the optimised  
253 parameters obtained from the fitting of NO<sub>2</sub> data. Finally, the uncertainties in the Gaussian plume estimates are obtained by  
254 propagation of the uncertainties in the fitted parameters for the wind speed and for the source strength.

255 To ensure the convergence of the minimization algorithm, first-guessed values of the fitted parameters need to be  
256 carefully prescribed: parameters of the centre-line curves, for example, are initialised from the curves retrieved by the plume  
257 detection algorithm, and the initial wind speed is calculated as in the CSF method (see Sect. 2.1.1). Most importantly, the  
258 prior values of emission parameters are set to the *true* summertime source emission strength. Thus, unlike any of the other  
259 methods studied in this work, the GP method integrates an important constraint on the emissions which implies that the  
260 estimated values, hence the method's performance, are not entirely determined by the information contained within the



261 synthetic satellite observations alone. This limitation should be taken into account when applying this method to invert from  
262 real satellite data emissions of sources whose amplitudes are barely known.

#### 263 **2.1.4 Integrated mass enhancement (IME) method**

264 The IME method integrates the total mass enhancements of CO<sub>2</sub> or NO<sub>2</sub> above the background that can be associated with  
265 detectable plumes. Then, following Frankenberg et al. (2016), the relationship between IMEs and emissions (Q) can be  
266 approximated by a linear relationship defined by the residence times ( $\tau$ ) of the species within the plumes (Eq. 1):

$$Q = \frac{1}{\tau} IME \quad (1)$$

$$\tau = \frac{U_{eff}}{L} \quad (2)$$

267 The residence time can in turn be expressed as a characteristic plume length  $L$  divided an effective wind speed  $U_{eff}$  (Eq.  
268 2). For example, Varon et al. (2018), who applied the IME method with CH<sub>4</sub> observations, derived  $U_{eff}$  from 10 m wind  
269 speeds using large eddy simulations (LES). Here, the plume detection algorithm which identifies either CO<sub>2</sub> or NO<sub>2</sub>  
270 enhancements from the background is the same as the one used by the CSF and GP methods, but the detected area of the  
271 plume over which the integration is performed is dilated using a circular kernel in order to increase the number of integrated  
272 pixels (Hakkarainen et al., 2023b). Missing values are filled using a normalised convolution and estimates are rejected when  
273 less than 75% of valid pixels are available for the detected plume. The characteristic length  $L$  is computed from the centre-  
274 line of the plume as the arc length to the most distant detected pixel minus 10 km, but at least 10 km. Moreover, the effective  
275 wind speed  $U_{eff}$  is extracted by using the same vertically weighted average as the CSF method. If NO<sub>2</sub> observations are used  
276 in conjunction with CO<sub>2</sub> observations, the integration area is established by the application of the plume detection algorithm  
277 with NO<sub>2</sub> data. Then, to estimate CO<sub>2</sub> emissions, the IME is calculated over this area with CO<sub>2</sub> observations. Finally, the  
278 uncertainty in the IME estimates is computed by propagation of uncertainty from the single sounding precision of satellite  
279 data and an estimate of the uncertainty in the wind speed.

#### 280 **2.1.5 Divergence method**

281 The divergence method, initially introduced by Beirle et al. (2019, 2021), was used to estimate NO<sub>x</sub> emissions based on  
282 TROPOMI NO<sub>2</sub> observations. For this study, the method has been modified in order to estimate CO<sub>2</sub> emissions, as outlined  
283 in Hakkarainen et al. (2022) where a detailed theoretical analysis of this approach can be found in the supplementary  
284 material. The divergence method is based on the continuity equation at steady state (Jacob, 1999), where the divergence of a  
285 vector field  $F$  (flux) is defined as the difference between emissions  $E$  and sinks  $S$  (Eq. 3):

$$\nabla \cdot F = E - S \quad (3)$$

$$F = (F_x, F_y) = (\Delta I \cdot U_{eff}, \Delta I \cdot V_{eff}) \quad (4)$$

288 Since CO<sub>2</sub> lifetime is extremely long, the sink term can be neglected. However, before applying the divergence operator to  
289 XCO<sub>2</sub> images, the atmospheric background needs to be removed in order to extract purely the XCO<sub>2</sub> enhancements. For this  
290 purpose, a median filter is applied to the data and the resulting field is subtracted from the original data. Moreover, in order  
291 to improve the accuracy of the estimates when CO<sub>2</sub> noise levels are high, data first undergo a denoising process using a 5×5  
292 pixel mean filter. The flux field  $F$  is then defined at each pixel by the Eq. 4 where  $\Delta I$  is the vertical column density  
293 enhancement above background, and  $U_{eff}$  and  $V_{eff}$  are the eastward and northward winds, respectively, interpolated at the  
294 location of the pixel and at the time of the satellite observations, and vertically averaged using the GNFR-A/SNAP-1  
295 emission profile (Brunner et al., 2019).

296 Divergence maps are computed from the mass flux field using a finite difference approximation. The divergence map is  
297 then averaged over a long period to enhance the emission signal, while reducing the impact of noise and the spatio-temporal  
298 variations of the CO<sub>2</sub> background. Here, divergence maps are averaged over one year. In theory, the divergence method can  
299 also be used to estimate emissions from single-overpass images such as the cross-sectional flux method (as the two methods  
300 are in theory similar, see Koene et al. 2024). However, we choose in this study to focus on the standard application of this  
301 method (e.g., Beirle et al. 2019, 2021, 2023; Hakkarainen et al., 2022, Sun et al., 2022), which provides temporally averaged  
302 estimates. Appendix A provides a brief overview of the performance when estimating emissions from individual images with  
303 different versions of the divergence approach.

304 For a specific source, the annual estimate of the emissions is then computed from the enhancement in the averaged  
305 divergence field by using a peak fitting approach which fits the divergence map by a function including a Gaussian and a  
306 linear term centred at the source (Beirle et al, 2021). Emissions, and more generally the parameters, of the peak function are  
307 determined by an adaptive Markov chain Monte Carlo (MCMC) that also provides the uncertainties in the estimates from the  
308 standard deviations of the sampled posterior distributions of the parameters.

## 309 **2.2. Synthetic satellite observations of CO<sub>2</sub> and NO<sub>2</sub>**

310 In this study, synthetic satellite observations of CO<sub>2</sub> and NO<sub>2</sub> were generated from atmospheric simulations in order to  
311 evaluate and compare the ability of the methods described in Sect. 2.1 for retrieving CO<sub>2</sub> or NO<sub>2</sub> emissions from point  
312 sources or urban areas using satellite imagery akin to that provided by the upcoming CO2M mission. These simulated  
313 satellite data are readable by the ddeq Python library and were produced as part of the SMARTCARB project and have been  
314 extensively described and used in previous works (e.g. Brunner et al., 2019; Kuhlmann et al., 2019; 2020; 2021). They are  
315 openly accessible from <https://doi.org/10.5281/zenodo.4048227> (Kuhlmann et al., 2020b).

316 Atmospheric concentrations of CO<sub>2</sub> and NO<sub>2</sub> were simulated by the COSMO-GHG atmospheric transport model (Jähn et  
317 al., 2020) with a vertical resolution of 60 levels up to an altitude of 24 km and with a horizontal resolution of about 1 km × 1  
318 km for a domain centred over the city of Berlin. The domain extends about 750 km in the east-west and 650 km in the south-  
319 north direction. Simulations provided hourly outputs for nearly the entire year 2015. In order to generate realistic  
320 simulations, initial and lateral boundary conditions for meteorological variables and tracers were extracted from products of

321 the European Centre for Medium-Range Weather Forecasts (ECMWF) and MeteoSwiss (Kuhlmann et al., 2019).  
322 Furthermore, CO<sub>2</sub> emissions included both the anthropogenic and biospheric components which were interpolated onto the  
323 COSMO grid at a temporal resolution of one hour: anthropogenic emissions were largely derived from the TNO/MACC-3  
324 inventory (Kuenen et al., 2014) and biospheric fluxes were simulated with the Vegetation Photosynthesis and Respiration  
325 Model (VPRM, Mahadevan et al., 2008). NO<sub>x</sub> emissions were also derived from the TNO-MACC-3 inventory and  
326 atmospheric simulations used a simplified NO<sub>x</sub> chemistry with a fixed NO<sub>x</sub> decay time of 4 hours. NO<sub>x</sub> concentrations were  
327 converted to NO<sub>2</sub> concentrations using an empirical equation for the evolution of NO<sub>2</sub> : NO<sub>x</sub> ratios downwind of emission  
328 sources (Düring et al., 2011).

329 To generate synthetic satellite observations similar to CO2M observations, the XCO<sub>2</sub> and NO<sub>2</sub> column densities derived  
330 from the COSMO-GHG simulations were sampled at the resolution of 2 km × 2 km along 250 km wide satellite tracks  
331 (Kuhlmann et al., 2019); these tracks were computed using an orbit simulator and correspond to a hypothetical constellation  
332 of six CO2M satellites. In addition to XCO<sub>2</sub> and NO<sub>2</sub> column-average data, a cloud mask was generated from the total cloud  
333 fraction computed by the COSMO-GHG model. For CO<sub>2</sub> data, all pixels with cloud fraction larger than 1% were removed as  
334 CO<sub>2</sub> retrievals are strongly impacted by clouds (Taylor et al., 2016). For NO<sub>2</sub> data, less sensitive to clouds, a threshold of  
335 30% on the cloud fraction was used to select valid pixels (e.g. Boersma et al., 2011). Figure 2 illustrates a COSMO-GHG  
336 simulation of XCO<sub>2</sub> over the SMARTCARB domain, on which are represented synthetic XCO<sub>2</sub> data corresponding to a  
337 CO2M satellite overpass.

338 For the purposes of this benchmarking study, we use the configuration of the SMARTCARB dataset where the CO2M  
339 constellation consists of three satellites. By choosing this, we follow the recommendation of Kuhlmann et al. (2021) that a  
340 constellation of at least three CO2M satellites is necessary for a proper estimation of the annual emissions from weak  
341 sources and in regions such as central Europe where cloud cover dramatically reduces the number of estimates. When  
342 ignoring clouds, this constellation of three satellites leads to observing each local source within the SMARTCARB domain  
343 once every other day; if we consider that a satellite image is usable if there are at least 50 data pixels next and downwind to  
344 the source, then we can use about 3000 images to determine the emissions of the 16 local sources considered in this study.  
345 But, if we consider the cloud cover, only 500 images remain usable.

346 The characteristics of the uncertainties in the synthetic CO2M observations were computed using three different  
347 uncertainty scenarios (low, medium, high). Simulated XCO<sub>2</sub> column densities were thus assigned random errors by  
348 employing various levels of instrumental noise in the error parameterization formula. This formula, used for generating the  
349 errors, takes into account the Solar Zenith Angle (SZA) and surface albedos (Buchwitz et al., 2013). The NO<sub>2</sub> column  
350 densities were assumed to be characterised by random uncertainties of different constant values depending on the chosen  
351 uncertainty scenario. These values are defined for clear sky conditions and increase in the presence of clouds; nearly  
352 doubling for a cloud fraction of 30%. No systematic errors were prescribed for either XCO<sub>2</sub> or NO<sub>2</sub> column averaged data. In  
353 this study, the characteristics of the random uncertainties prescribed to the synthetic data are chosen according to the  
354 requirements of the CO2M mission (Meijer et al., 2019). For XCO<sub>2</sub> retrievals, random errors are generated using the error

355 parameterization formula with a single sounding precision of 0.7 ppm for vegetation albedos and a SZA of 50°. For NO<sub>2</sub>  
356 retrievals, a single sounding precision in cloud-free conditions of  $2 \times 10^{15}$  molecules cm<sup>-2</sup> is prescribed.

### 357 **2.3. Benchmarking scenarios**

358 The relative performance of the different inversion methods to estimate CO<sub>2</sub> emissions are evaluated for the 15 strongest  
359 point sources of the SMARTCARB domain and for the city of Berlin (Fig. 2 and Table 1 in Kuhlmann et al., 2021). These  
360 16 sources cover a large emission range that extends from 3.7 MtCO<sub>2</sub>.yr<sup>-1</sup> for the power plant located in Chvaletice (CZ) to  
361 40.3 MtCO<sub>2</sub>.yr<sup>-1</sup> for the power plant located in Jänschwalde (DE); these values being the annual mean emissions at the time  
362 of the satellite overpass (10:30 UTC) used in the COSMO-GHG simulations. It is worth mentioning that the distribution of  
363 the source emissions is skewed towards the lowest value as the median emission rate in the collection is around 9.6  
364 MtCO<sub>2</sub>.yr<sup>-1</sup> and 75% of the sources emit less than 14 MtCO<sub>2</sub>.yr<sup>-1</sup>.

365 In order to thoroughly evaluate the relative performance of the different methods and the sensitivity of these  
366 performances to different factors, the benchmarking study is carried out according to several scenarios that share the same  
367 features for the simulated data and for the source collection that have been described above. The most optimistic or ideal  
368 scenario corresponds to the application of inversions to CO<sub>2</sub> and NO<sub>2</sub> images without the removal of pixels associated to  
369 cloud-cover (ignoring the clouds modelled with the COSMO-GHG model; we label such inversions “cloud-free” hereafter)  
370 and with a perfect knowledge of the wind field (i.e. using directly the winds from the COSMO-GHG model, denoted  
371 SMARTCARB winds). It is the ideal case because 1) the joint analysis of NO<sub>2</sub> and CO<sub>2</sub> images strengthen the estimates  
372 compared to the analysis of CO<sub>2</sub> images only; 2) ignoring the potential loss of data due to cloud cover in the CO<sub>2</sub> and NO<sub>2</sub>  
373 images yield full images, whose analysis is more robust than that of partial images, and thus provides a higher number and  
374 precision of estimates. The results derived from this benchmarking scenario should be seen as an upper limit of what the  
375 inversion methods could achieve in terms of accuracy and number of estimates. The most realistic scenarios take cloud cover  
376 into account and use winds extracted from the ERA5 wind product (Hersbach et al., 2020) that is independent from the  
377 inverted data and whose resolution (~0.25°) is much coarser than that of the SMARTCARB winds (~0.01°). The results  
378 derived from this benchmarking scenario should be seen as a lower limit for the method's performance.

379 The differences between the ERA5 and SMARTCARB wind products are significant at the 16 sources considered in this  
380 study: the annual mean biases between these two wind products in 2015 range from 0.1 ms<sup>-1</sup> to 1.5 ms<sup>-1</sup> depending on the  
381 source with an average value across the sources of 0.6 ms<sup>-1</sup> while RMSEs range from 1.1 ms<sup>-1</sup> to 2.1 ms<sup>-1</sup> depending on the  
382 source with an average value across the sources of 1.5 ms<sup>-1</sup> (Fig. A2). The biases per source are systematically positive since  
383 SMARTCARB tends to provide larger winds than ERA5. With such differences, comparing scenarios with the same  
384 characteristics but using different wind products allows us to gain insight into the method's sensitivity to wind uncertainties.  
385 Additional benchmarking scenarios were designed to test the sensitivity of the methods with respect to other factors,  
386 including the consideration of cloud cover in satellite data and the use of NO<sub>2</sub> for plume detection and characterization. All  
387 benchmarking scenarios are listed in Table 2.

## 388 2.4. Benchmarking metrics

389 For a given benchmarking scenario, the performances of the different inversion methods can be evaluated through the  
390 number of single-image estimates that can be retrieved regarding the number of available satellite images: ~500 or ~3000  
391 considering or ignoring the cloud cover in the data. Performances can be assessed as well through the quality of the  
392 estimates; the accuracies of the methods are then assessed by comparing the estimates retrieved from single satellite  
393 overpasses to the corresponding *true* values that were used to generate the synthetic satellite data. More precisely, inversion  
394 results are analysed in terms of distributions of the differences between the estimated and the true emissions of all the  
395 sources considered in this study. We will refer to these differences in the following as *deviations*. More precisely, our  
396 analysis will mostly focus on examining the distributions of the *relative* deviations, i.e. the differences between estimated  
397 and true emissions divided by the true emissions, in order to fairly compare results across sources with significantly different  
398 magnitudes (Sect. 2.3). Furthermore, to properly describe distributions that may be very different from Gaussian  
399 distributions, box plots are used, in which the median values, the interquartile ranges (IQRs), the 10<sup>th</sup> and the 90<sup>th</sup> percentiles  
400 of the distributions are represented.

401 The ability of the different inversion methods to estimate source emissions can also be analysed from the study of the  
402 annual or monthly averages of the single-image estimates. Benchmarking results are then evaluated for each source in terms  
403 of relative deviations of the annual/monthly estimates from the annual/monthly true emissions and, in terms of Root Mean  
404 Square Errors (RMSE) in order to provide a global indicator for the accuracy of the annual/monthly estimates across all  
405 sources.

406 In this study, the annual/monthly averages of the single-image estimates for a given source are computed using three  
407 different methods which are 1) the arithmetic means of all the single-image estimates of the source emission that have been  
408 generated from inverting one year/month of data, 2) the means of these estimates weighted by the inverse of their computed  
409 variances (Sect. 2.1) and 3) the medians of these estimates. The annual/monthly inverse variance weighted means  
410 incorporate the information provided by the methods on the quality of the estimates when averaging, whereas the  
411 annual/monthly medians are statistical indicators that are more robust to outliers than the means. Moreover, since the Div  
412 method is applied by temporally averaging satellite observations over the year, it produces only a single annual estimate for  
413 each source; we will thus consider that the three types of annual/monthly estimates are all equal to this single estimate.

414 It is important to note that the annual and monthly estimates are affected by temporal sampling biases when inversion  
415 methods use data filtered by cloud cover. Specifically, the presence of denser cloud cover during winter generally results in  
416 over-representation of emission estimates during summer and hence could lead to an underestimation of annual estimates as  
417 emissions are higher during winter due to increased fossil fuel consumption associated with electricity and heat production.  
418 Although more advanced methods, such as fitting periodic curves to capture seasonal cycles as demonstrated by Kuhlmann  
419 et al. (2021) could potentially enhance the accuracy of estimates, they are not included in this study. However, these

420 temporal sampling biases are integrated in the results as the annual/monthly estimates are compared to the true  
421 annual/monthly emissions which are computed by considering all the days of the year/months.

### 422 **3 Results on emission estimates based on individual images**

423 The following subsections present a comparative study of the CSF, GP, IME, and LCSF methods for estimating emissions  
424 from single images. In the following, we will refer to these kinds of estimates as *single-image* estimates. Note that, as the  
425 methods use different algorithms for plume detection and emission quantification, which include different rejection criteria  
426 (Sect. 2.1), they produce different sets of estimates.

#### 427 **3.1 Sensitivity to the emission strengths of the sources**

428 In the optimal scenario (cloud-free, SMARTCARB winds, CO<sub>2</sub> and NO<sub>2</sub> data), all methods tend to provide more accurate  
429 estimates for strong sources than for weak sources, and this trend is particularly noticeable for the IME and CSF methods  
430 (Fig. 3). The median values of the absolute relative deviations for weak sources (emissions ranging from 0 to 6.9 MtCO<sub>2</sub>/yr  
431 in the 1<sup>st</sup> row of Fig. 3) are 207% (IME method) and 54% (CSF method), respectively. In contrast, for strong sources  
432 (emissions ranging from 15.6 to 53.2 MtCO<sub>2</sub>/yr in the 4<sup>th</sup> row of Fig. 3), they are approximately 47% (IME) and 28% (CSF),  
433 respectively. The inversion methods are also more prone to produce unrealistic values for weak sources as the distributions  
434 are strongly skewed for this type of sources: the 95<sup>th</sup> percentile accuracy indicator is indeed 1128%, 584%, 172% and 178%  
435 for the IME, CSF, GP and LCSF inversion models respectively (1<sup>st</sup> row in Fig. 3). For strong sources, this indicator is  
436 significantly lower, decreasing to 200%, 108%, 90% and 76%, respectively (4<sup>th</sup> row in Fig. 3). Atmospheric signals  
437 generated by strong sources are more distinct from the background than those from weak sources and as a result, the signal-  
438 to-noise ratio in the XCO<sub>2</sub> and NO<sub>2</sub> images is better which helps to reduce uncertainties in the determination of their  
439 emissions. For low-emitting sources, the performance of the inversion methods can be degraded by the limited number of  
440 enhanced pixels that are detected in images with noise; this limitation makes the identification of plume centre-lines by the  
441 CSF, IME and GP methods challenging (Sect. 2.1). This problem could have impacted the GP method, but its current  
442 implementation incorporates prior knowledge filtering out estimates that fall outside the 25% to 400% range from the prior.  
443 This filtering process is expected to improve the accuracy of the GP method, especially for weak sources.

444 Biases in the emission estimates may also depend on the strength of the source, as observed in the IME and CSF methods  
445 which strongly overestimate the emissions of weak sources compared to strong sources. For weak sources, the median of the  
446 deviation distributions for the IME and CSF models (blue bars, 1<sup>st</sup> row of Fig. 3) are +116% and +50%, respectively,  
447 compared to +16% and +11% for strong sources (blue bars, 4<sup>th</sup> row of Fig. 3). This discrepancy is probably due to the plume  
448 detection algorithm, which, for weak sources, may wrongly attribute enhancements from other sources in the vicinity of the  
449 source of interest and thus artificially increase the amplitude of the detected emissions. Conversely, the LCSF approach  
450 tends to underestimate the emissions of strong sources while slightly overestimating those of weak sources, with the median

451 of the deviation distribution being  $-26\%$  (blue bar, 4<sup>th</sup> row of Fig. 3) and  $+12\%$  (blue bar, 1<sup>st</sup> row of Fig. 3) respectively. The  
452 underestimation of source emissions could be attributed to a tendency of the method to overestimate the amplitudes of the  
453 background for non-isolated sources: contrary to the other methods, the LCSF method does not remove the influence of  
454 neighbouring plumes when computing the background around a given source. Another explanation could lie in the fact that  
455 this method uses 100-m winds as effective winds while, especially for strong emitting sources, these winds are lower than  
456 the GNFR-A average winds used by the other methods.

### 457 **3.2 Impact of the use of NO<sub>2</sub> images for the detection of plumes**

458 The use of NO<sub>2</sub> data to identify and characterise plumes increases the number of estimates for all inversion methods  
459 compared to CO<sub>2</sub>-only inversions, as shown in Figure 4 (blue vs orange bars). The increase is significant for the IME and GP  
460 methods ( $\sim 93\%$  and  $\sim 70\%$ ), moderate for the CSF method ( $\sim 34\%$ ), and slight for the LCSF method ( $\sim 4\%$ ). The IME, GP,  
461 and CSF methods rely on a plume detection algorithm that is less reliable when using only CO<sub>2</sub> observations (Kuhlmann et  
462 al. 2019). Of these three, the CSF method requires fewer pixels to detect and quantify plumes, resulting in a larger proportion  
463 of still quantified plume cases than the IME and GP methods when having CO<sub>2</sub> data only. The detection of plumes by the  
464 LCSF method is performed on data slices whose pixels are relatively close to sources and where XCO<sub>2</sub> enhancement signals  
465 due to emissions are thus relatively strong; this may explain the only small benefit for this method of using joint CO<sub>2</sub> and  
466 NO<sub>2</sub> images to better determine the shape of the plumes.

467 When using CO<sub>2</sub> and NO<sub>2</sub> data, the maximum number of estimates obtained from each inversion method varies  
468 significantly: the IME method produces the smallest number of estimates, with 1661, while the LCSF method produces the  
469 largest, with 2722. The GP and CSF methods, based on the same algorithm of plume detection as the IME method, produce  
470 up to 1776 and 2012 estimates, respectively. These differences can be attributed to the differences in the number of detected  
471 pixels below which the algorithm rejects plumes and, in the emission quantification algorithms used by the different  
472 methods. In addition, the overall complexity of the IME, CSF and GP methods, which use a relatively large number of  
473 rejection criteria likely explains why these three methods deliver much fewer estimates than the LCSF method. The relative  
474 efficiency and robustness of the plume detection algorithm of the LCSF method is evidenced when using CO<sub>2</sub> data only to  
475 determine emissions: the number and accuracy of estimates is hardly changed compared to the inversions performed with  
476 CO<sub>2</sub> and NO<sub>2</sub> data; contrarily to the other methods whose algorithms are more sensitive to uncertainties in XCO<sub>2</sub> data and  
477 which need NO<sub>2</sub> data to accurately fit a plume coordinate system to the data.

478 The inclusion of NO<sub>2</sub> data does not appear to significantly improve the overall performance of the GP and LCSF methods  
479 in terms of accuracy of the CO<sub>2</sub> emission estimates (lower panel in Fig. 4). However, for the LCSF method, there is a notable  
480 reduction in the 95<sup>th</sup> percentile of the relative absolute deviations from 175% without NO<sub>2</sub> to 115% with NO<sub>2</sub>. For the CSF  
481 method, the use of NO<sub>2</sub> data strongly improves its overall performance as the 3<sup>rd</sup> quartile and the median of the absolute  
482 residuals are for example significantly decreased, from  $\sim 127\%$  down to  $\sim 74\%$  and from  $\sim 54\%$  to  $\sim 36\%$ , respectively. As the  
483 CSF method rejects fewer estimates when using CO<sub>2</sub> data only than the GP method, its accuracy decreases because with a

484 more permissive filtering, it may include complex cases for which emissions are difficult to estimate. This may also explain  
485 why the CSF estimates are less biased, with a significantly lower median relative deviation, in cases where inversions also  
486 use NO<sub>2</sub> data (upper panel in Fig. 4).

487 In contrast, the precision of the IME method decreases when using NO<sub>2</sub> data, but this fact could be related to a numerical  
488 artefact: the IME method performs much better for high-emitting sources than for low-emitting sources (see Sect. 3.1) and  
489 the use of NO<sub>2</sub> data likely allows constraining small sources more efficiently than with CO<sub>2</sub> data only. Therefore, when  
490 adding NO<sub>2</sub> data, the number of low-emitting sources which are estimated increases more than for the high-emitting sources  
491 and then the overall performance degrades. This bias associated to the relative bad estimation of low-emitting sources is  
492 confirmed when deviations are used to assess performance instead of relative deviations: the absolute deviations associated  
493 to the IME estimates globally decrease with the use of NO<sub>2</sub> data with for example the median error decreasing from ~15 to  
494 ~11.5 MtCO<sub>2</sub>/yr.

### 495 **3.3 Impact of the cloud cover**

496 The impact of clouds is studied by comparing inversions with cloud-free images to inversions with cloud-filtered images  
497 (Sect. 2.3). When disregarding cloudy pixels in the XCO<sub>2</sub> and column-averaged NO<sub>2</sub> data, the number of estimates from all  
498 the methods is considerably reduced, with a decrease of 94%, 85%, 85% and 88% for the IME, CSF, GP and LCSF methods  
499 respectively (Table 3). The number of estimates that can be provided for the cloud-filtered configuration with  
500 SMARTCARB winds is at the maximum equal to 313 (LCSF) and decreases to 96 for the IME method which can provide  
501 robust estimates for images free of clouds only as this method requires integrating enhancements over the full extent of  
502 plumes. As sources are characterized by different cloud covers, the number of estimates per year and per source ranges from  
503 1 to 12 (IME), from 6 to 28 (CSF), from 8 to 23 (GP) and from 15 to 26 (LCSF).

504 Furthermore, the filtering of data pixels removing those with a significant cloud cover not only affects the number of  
505 estimates but also impacts the performance of the methods, although to a much lesser extent. When comparing results  
506 obtained from the same images, cloud-free inversions produce slightly better results than cloud-filtered inversions (Fig A3).  
507 This is because, in images partially masked by cloud cover, some pixels containing useful information are likely removed,  
508 which can lead to less accurate determination of emissions. Consistently, if the threshold of cloud cover above which XCO<sub>2</sub>  
509 images are discarded for the analysis is increased from 1% to 2% or 5%, the performance of the methods does not  
510 significantly increase, unlike the number of estimates, which can increase, e.g. by 12% and 29% respectively when using the  
511 LCSF method (Fig. A4).

### 512 **3.4 Impact of uncertainty in the wind**

513 As mentioned above, in order to assess the impact of potential uncertainties in the wind, a series of inversions is carried out  
514 with a different wind product than the one used to generate the synthetic XCO<sub>2</sub> and NO<sub>2</sub> data. For this purpose, the  
515 SMARTCARB winds are replaced by ERA5 winds and the differences between these two wind products are characterised at



516 the sites of this study by random and systematic components (Sect 2.3 and Fig. A3). Notably, ERA5 winds show  
517 systematically lower values.

518 For all inversion methods, the global accuracies of the estimates, evaluated in terms of relative absolute deviations, are  
519 only slightly reduced when using ERA5 winds instead of SMARTCARB winds (lower panel in Fig. 4, green vs red bars).  
520 There are a few possible explanations for this: the temporal or spatial uncertainties in wind components are only a minor  
521 source of uncertainty compared to other factors impacting the determination of the estimates by the different inversion  
522 methods such as, for example, uncertainties in the XCO<sub>2</sub> and NO<sub>2</sub> columns densities (Sect. 2.2) or over-simplified  
523 assumptions in plume detection or quantification algorithms. Kuhlmann et al. (2020, 2021) showed, for instance, that the  
524 determination of the CO<sub>2</sub> background field could introduce significant uncertainties in the estimates. Furthermore, as  
525 indicated by Reuter et al. (2019), one of the important benefits of satellite imagery is that uncertainties related to  
526 meteorological variables likely average out when emission estimates are sampled along significant areas of plumes.

527 However, the fact that ERA5 wind values are systematically lower than those of SMARTCARB winds has an impact on  
528 the median values of the relative deviations, i.e. on the biases in the estimates. While the accuracies in terms of relative  
529 absolute deviations are slightly affected by using either wind product (bottom panel in Fig. 4, green vs red bars), biases can  
530 be significantly increased, as in the cases of the GP and LCSF methods whose estimates are on average underestimated if  
531 inversions use ERA5 winds instead of SMARTCARB winds. The lower amplitudes of the ERA5 winds explains also that the  
532 results for the IME and CSF methods improve, especially for the 95<sup>th</sup> percentiles of the absolute deviation distributions  
533 which respectively decrease from around 504% and 411% to 370% and 286% respectively. The systematic overestimation of  
534 the estimates evidenced above for the CSF and the IME methods is therefore mitigated when using ERA5 winds (top panel  
535 in Fig. 4).

536 As mentioned previously (Sect. 2.3), the benchmarking scenario for which inversions are performed with ERA5 winds  
537 and data filtered for cloud cover, is the closest to real conditions of monitoring emissions from data images delivered by  
538 satellites. For this scenario with CO<sub>2</sub> and NO<sub>2</sub> data, the GP and LCSF methods show the best performances in terms of  
539 global accuracies with respectively IQRs of 25–62% and 17–55% for the distributions of the absolute relative deviations (red  
540 boxes in Fig. 4). It is interesting to note that the overall accuracies of these methods are similar for this realistic scenario and  
541 the ideal scenario where inversions are performed with cloud-free data and SMARTCARB winds. Contrarily, the number of  
542 estimates strongly decreases when inversions are performed with cloud-filtered data such as, for example, from 2722 to 318  
543 estimates for the LCSF method (see Table 3).

## 544 **4 Results on annual and monthly averages of the emissions**

### 545 **4.1 Annual estimates**

546 To evaluate how well an inversion method performs on an annual basis, we include all image estimates generated by the  
547 method, regardless of their uncertainty. We calculate annual estimates for a given source using three methods, as described

548 in Sect. 2.4: 1) by taking the average of all available image estimates for the source over the entire year, 2) by taking the  
549 weighted average of these image estimates based on their uncertainty, and 3) by taking the median value of these image  
550 estimates. Because the Div method only provides one estimate per year, its annual estimates are the same, irrespective of the  
551 calculation method used. In order to compare for a given source the three estimated annual values to the true emission, we  
552 define this latter as the arithmetic mean of the true emissions values for the source over all 365 days of the year.

553 When annual estimates are calculated as arithmetic means or medians of individual image estimates, the GP and LCSF  
554 methods generally outperform the other methods. Indeed, for cloud-free inversions with CO<sub>2</sub> and NO<sub>2</sub> data, the median  
555 deviations for the annual arithmetic means (solid lines, 2nd column of Fig. 5) are 8% (GP), 14% (LCSF), 73% (IME), 35%  
556 (CSF), and 64% (Div), and the median deviations for the annual medians (dotted lines, 2nd column of Fig. 5) are 14% (GP),  
557 21% (LCSF), 54% (IME), 13% (CSF), and 64% (Div). However, if annual estimates are calculated as the means of image  
558 estimates weighted by their uncertainty, the relative performance of the methods changes. In this case, the median deviations  
559 for annual weighted means (dashed lines, 2<sup>nd</sup> column of Fig. 5) are 28% (GP), 48% (LCSF), 46% (IME), and 12% (CSF).  
560 Thus, using weighted means to calculate annual estimates significantly improves, especially for low-emitting sources, the  
561 performance of the IME and CSF methods while having a negative impact on the GP and LCSF methods. This finding  
562 indicates the reliability of the uncertainties in the estimates produced by the IME and CSF methods compared to the other  
563 methods and, if we use weighted means to compute annual estimates, the accuracies of the IME and CSF methods increase  
564 significantly.

565 Figure 6 displays the inversion results for the annual estimates in a different but complementary way compared to Fig. 5:  
566 the estimated annual emissions are represented with respect to the true ones which in particular allows illustrating whether  
567 annual estimates are over- or under-estimated for a certain type of source and by a given inversion method. In order to  
568 consider the best performance for each method according to what has been shown above, annual estimates represented in the  
569 figure, and used for the analysis of the results made below, are arithmetic means of single-image estimates for the LCSF and  
570 the GP methods, while they are weighted means for the IME and CSF methods. Furthermore, Fig. 6 illustrates more clearly  
571 than Fig. 5 the fact that, when weighted averages are used as annual estimates, the latter methods produce annual estimates  
572 whose precision is comparable for weak *and* strong sources while the global precision of estimates derived from single  
573 images by these methods is significantly lower for weak sources (Fig. 3); averaging single-image estimates weighted by their  
574 uncertainty thus strongly increases the performance of the IME and CSF methods at the annual scale for low-emitting  
575 sources. However, even though the amplitudes of the relative deviations are similar between strong and weak sources, they  
576 have opposite signs: annual estimates for strong sources are generally underestimated while annual estimates for weak  
577 sources are generally overestimated.

578 Contrary to the results for the estimates retrieved from single images (Fig. 4), the CSF, GP and LCSF approaches show  
579 similar performance, with a slight advantage for the GP method, when estimating annual emissions if we consider the  
580 ensemble of the benchmarking scenarios. For example, in the case of inversions from cloud-filtered CO<sub>2</sub> and NO<sub>2</sub> data and,  
581 with SMARTCARB/ERA5 winds, the relative RMSEs are 18/27% (CSF), 20/20% (GP) and 17/31% (LCSF). The analysis

582 of Fig. 3 shows that the LCSF method produces single-image estimates that are slightly more accurate but more biased than  
583 that of the GP method. Thus, the compensation of errors when averaging single-image estimates over a year may be less  
584 effective for the LCSF method than for the GP method leading to similar global accuracies for both methods. For instance,  
585 the LCSF method has a greater tendency to underestimate high emissions (4<sup>th</sup> row of Fig. 3) which likely explain why,  
586 contrarily to the GP method, it systematically underestimates the emissions of the strong emitting power plant located in  
587 Jämschalde, regardless of the inversion scenario (Fig. 6). With respect to its results for single-image estimates, the CSF  
588 method has significantly better results at the annual scale when annual estimates are computed as weighted averages of  
589 single-image estimates.

590 Even when annual estimates are computed for the IME method as weighted averages of the single-image estimates, this  
591 method still show smaller accuracies compared to the CSF, GP and LCSF methods: the median values of the deviations for  
592 the annual estimates are for example 39% (IME), 20% (CSF), 11% (GP) and 21% (LCSF) when considering the best scores  
593 for the inversions performed with ERA5 winds and cloud-filtered data (4<sup>th</sup> column of Fig. 5). The relative performance of the  
594 IME method is even worse when analysing the performance in terms of RMSE because, despite a weighting of estimates  
595 according to their quality or uncertainty in the annual averages, this method produces for some sources annual estimates that  
596 strongly deviate from the actual values, as in the cases of Boxberg or Schwarze Pumpe power plants (Fig. 6). Moreover, the  
597 deviations of the Div method compared to that of the CSF, GP and LCSF methods are higher for most of sources except for  
598 strong sources (true annual emissions > 15 MtCO<sub>2</sub>/yr) when inversions are performed using cloud-filtered data and ERA5  
599 winds (4<sup>th</sup> column of Fig. 5).

600 It is noteworthy that annual estimates for most inversion methods are comparable between inversions using data with or  
601 without clouds (comparison between the 2<sup>nd</sup> and 3<sup>rd</sup> columns, Fig. 5), and surprisingly the deviations of the IME and Div  
602 approaches are even smaller for inversions with cloud-filtered data. Despite significant differences in the number of image  
603 estimates between those two (i.e., cloud-filtered and cloud-free) inversion configurations, annual estimates are *on average*  
604 slightly affected when cloud cover is considered in the data, at least for the year and sources examined in this study.  
605 However, even though the relatively small number of image estimates in the inversion configuration with clouds does not  
606 hinder most methods from determining annual emissions of most sources, discrepancies can be high for some sources when  
607 estimates do not sample correctly the entire year and thus introduce an important temporal bias. For example, the GP method  
608 mostly estimates emissions during summer for the Jämschalde power plant when it uses the cloud-filtered inversion setup,  
609 explaining the strong underestimation of the annual emission of this source compared to the cloud-free case (top-left vs  
610 bottom-left panel of Fig. 6); this explains additionally why the RMSE increases significantly for the GP method (from 13%  
611 to 20% when inversions use SMARTCARB winds) when the cloud cover limits the number of single-image estimates. The  
612 IME method is also impacted by this temporal bias when the number of estimates is too small to properly capture the  
613 seasonal cycle of the emissions, as in the case of the Boxberg power plant. Moreover, whatever the benchmarking scenario,  
614 most inversion methods produce annual estimates for all the sources studied in this work, with the notable exception of the  
615 Div approach, which estimates annual emissions for only 10 out of 16 sources. This limitation, also present for cloud-free

616 data configurations, is related to the fact that some sources don't produce strong enough divergence peaks from which  
617 annual estimates can be made by this method.

618 As for the results concerning single-image estimates, the use of ERA5 winds instead of SMARTCARB winds has on  
619 average a very low impact on annual estimates delivered by the IME, CSF, GP and LCSF methods. For emissions estimated  
620 from cloud-free CO<sub>2</sub> and NO<sub>2</sub> data, the median deviations when inversions use SMARTCARB winds are indeed 46% (IME),  
621 12% (CSF), 8% (GP) and 14% (LCSF), and when inversions use ERA5 winds, they are equal to 46% (IME), 12% (CSF), 9%  
622 (GP) and 12% (LCSF) as shown in the comparison between the 2<sup>nd</sup> and 4<sup>th</sup> columns of Fig. 5. On the other hand, the overall  
623 accuracy of the Div method improves when inversions use ERA5 winds rather than SMARTCARB winds to estimate  
624 emissions. In this case, annual estimates are less prone to overestimation due to the generally lower amplitude of ERA5  
625 winds compared to SMARTCARB winds (Fig. A2). This also explains a stronger underestimation of the emissions of strong  
626 sources by the LCSF method, resulting in a decrease in the accuracy of the annual estimates for this kind of sources when  
627 this method uses ERA5 instead of SMARTCARB winds (left-bottom vs right-bottom panel of Fig. 6).

628 The overall precision of the annual estimates computed by the IME, CSF, GP and LCSF methods are, for all the  
629 benchmarking scenarios, significantly higher than the overall precision of their single-image estimates. For example, when  
630 inversions are performed with ERA5 winds and cloud-filtered data, which is the benchmarking scenario with the poorest  
631 results, the median deviations of the annual estimates are 39%, 20%, 11% and 21% whereas the median deviations of the  
632 single-image estimates are 73%, 35%, 46% and 37% for the IME, CSF, GP and LCSF methods. Despite the biases that can  
633 hamper the image estimates, the compensation for errors when averaging across a year allow to generate annual estimates  
634 that are more precise and this positive effect is amplified when error-weighted averages are used, as in the case of the IME  
635 and CSF methods.

#### 636 **4.2 Monthly estimates and seasonal cycle**

637 Monthly estimates can be computed using the same three methods as the annual estimates but, according to the results  
638 analysed in the former section, we choose to estimate monthly emissions with the method leading to the best performance at  
639 the annual scale: monthly estimates are thus calculated as the arithmetic means for the GP and LCSF methods and, as  
640 weighted means for the CSF and IME methods. Then, considering the distributions of image estimates month by month  
641 allows us to study how well inversion approaches capture the seasonal cycle of the true emissions. The analysis of Fig. 7  
642 shows however that none of them are able to do this when the cloudy pixels are masked: the seasonal cycle of the actual  
643 monthly emissions, i.e. maximal/minimal emissions for winter/summer months, is not reproduced by the inversion methods  
644 whose estimates are characterised by an erratic monthly evolution leading to inconsistent seasonal cycles. Even though a  
645 method correctly estimates annual emissions, some of its monthly estimates can be in important disagreement with the *true*  
646 monthly emissions as it is the case for the CSF method on the Heyden source or for the LCSF method on the Dolna Odra  
647 source (Fig. 7). Moreover, the methods generally fail to produce estimates for the winter months of the year due to the  
648 temporal sparsity of data when the impact of the cloud cover is taken into account.

649 If the number of estimates is higher, i.e. when clouds are not considered in the data, seasonal cycles derived from  
650 monthly estimates are in better agreement with that of the observations for most of inversion methods: the amplitude of the  
651 seasonal cycle of the data can be well reproduced as it is the case for the Janschwalde and Dolna Odra sources for example  
652 (Fig. A5). But, the averaged values of the seasonal cycles of the monthly estimates, i.e. the annual estimates, can still be in  
653 strong disagreement with that of the data even though the number of estimates is higher; this fact supports the presence of  
654 systematic biases in the estimates that was evidenced for most of the methods in the analysis of the results for single-image  
655 image estimates (Sect. 3.1).

## 656 **5 Discussion**

### 657 **5.1 Accuracy vs number of estimates**

658 For a given benchmarking scenario, the analysis conducted in Section 3 has evaluated the performance of the different  
659 methods in inferring estimates from individual images by considering all the estimates provided by each method for this  
660 scenario. In other terms, the analysis did not integrate any diagnostic regarding the quality of the estimates from these  
661 methods. However, we demonstrated in Sect. 4.1 that computing annual means of estimates weighted by their uncertainties  
662 can significantly improve the accuracy of the annual estimates when uncertainties are effectively characterised as in the case  
663 of the IME and CSF methods. Therefore, a study of the performance of inversion methods for estimating single-image  
664 estimates from synthetic XCO<sub>2</sub> images should as well integrate a characterization of the quality of its estimates. More  
665 precisely, different performance indicators or error estimates can be derived from the application of the inversion methods  
666 and such indicators can be used to identify and select the most reliable estimates. Nevertheless, there are no objective criteria  
667 to impose a threshold on the quality of the estimates; higher quality thresholds come with smaller sets of estimates, and  
668 optimal values depend on the inversion method. Indeed, not only do the different inversion methods calculate the  
669 uncertainties in the estimates in different ways but also the computed uncertainties only reflect part of the total/actual  
670 uncertainties, focusing on subsets of sources of uncertainties which differ across the different methods.

671 For a given inversion method, we attempt an effective quality indicator (QI) which would allow selecting estimates in a  
672 manner that the global accuracy of the method increases when the QI increases, and which would provide indications on the  
673 actual/total errors. We assume that the uncertainties in the estimates derived by the methods provide the best basis we can  
674 get from the algorithms described in Sect. 2.1 for the derivation of such an indicator. In principle, since dealing with sources  
675 of quantitatively different amplitudes (see Sect. 2.3) we should derive the QI in terms of *relative* uncertainties. And, if we  
676 define the QI as a threshold selecting the estimates whose relative uncertainties are below it, we should select the most  
677 reliable estimates regardless of the strength of the source they are associated with. However, this would be true if the  
678 methods perform independently with respect to the amplitudes of the emissions and this is not the case for most methods as  
679 illustrated in Sect 3.1. The CSF and IME methods for example strongly overestimate low-emitting sources compared to  
680 high-emitting sources which implies that the relative uncertainties of weak sources are underestimated by these methods

681 (Fig. 3). Therefore, if the threshold value of relative uncertainty was decreased, we would tend to select more bad than good  
682 estimates and the overall performance would decrease. Therefore, for these methods, we prefer to select estimates with  
683 respect to their uncertainties, and not to their *relative* uncertainties, which will mitigate the impact of the bias in the  
684 estimation of low-emitting sources.

685 In any case, determining whether a QI should be based on absolute or relative uncertainties depends on whether the  
686 overall performance of the method improves when estimates with decreasing absolute or relative uncertainties are chosen.  
687 Preliminary tests (not shown here) have established that the overall accuracy of the IME and CSF methods increases when  
688 the *absolute* uncertainty below which estimates are selected is decreased. For the GP and LCSF methods, this behaviour is  
689 obtained when *relative* uncertainties are used to discriminate estimates. Consistently, for all methods, the increase of  
690 performance is then associated with a reduction in the number of estimates and, in order to get a significant number of high-  
691 quality estimates, the value of uncertainty corresponding to the maximal accuracy of the method is arbitrarily set to the 10<sup>th</sup>  
692 percentile of the distribution of the absolute/relative uncertainties. Then, by varying its QI between this value and the  
693 maximal uncertainty of its estimates, each method can be thus associated to a range of accuracies with their respective  
694 number of estimates for a specific benchmarking scenario (e.g. cloud-filtered or cloud-free). In other words, inversion results  
695 can be represented by curves of accuracy *vs* number of estimates, which gives for each inversion method a complete  
696 overview of its performance in terms of accuracy and number of estimates.

697 To assess the inherent performance of the methods without considering the impact of the cloud cover or of the  
698 uncertainty in the winds, inversion results are analysed for the inversion configuration using XCO<sub>2</sub> and NO<sub>2</sub> cloud-free data  
699 and SMARTCARB winds, *i.e.* the same winds used to generate the synthetic XCO<sub>2</sub> and NO<sub>2</sub> observations. Figure 8  
700 illustrates that the overall accuracies of the CSF and IME methods are highly dependent on the selection of their estimates,  
701 and are therefore strongly correlated with their number of estimates. For instance, the IME and CSF methods exhibit large  
702 increases in the 3<sup>rd</sup> quartiles of their deviation distribution when the QIs of their estimates decrease: from 81% to 231%  
703 (IME) and from 43% to 75% (CSF) respectively. For these methods, the selection of estimates based on their quality  
704 indicators appears to be effective, as the 3<sup>rd</sup> quartiles and 95<sup>th</sup> percentiles, which indicate the proportion of poor estimates,  
705 significantly decrease with increasing quality index, *i.e.* with decreasing number of estimates. Therefore, the IME and CSF  
706 methods are very likely to produce reliable uncertainty estimates in the individual emission estimates and the definition and  
707 derivation of their QI reflect the level of accuracy of their estimates.

708 The LCSF and GP methods display a slight correlation between most of their accuracy indicators and the number of  
709 estimates. For instance, the 3<sup>rd</sup> quartiles of the distributions of relative absolute deviations remain relatively stable, varying  
710 only from 46% to 56% and from 51% to 59% for the LCSF and GP methods respectively, over their entire range of number  
711 of estimates. For these methods, the tradeoff between precision and number of estimates is not a critical issue and retrieving  
712 an important number of estimates does not imply a significant deterioration in accuracy. On the other hand, this also  
713 indicates that the current quality indicators for the GP and LCSF methods do not reflect the total/actual uncertainties in their  
714 estimates.

715 As the methods present different sensitivities of the accuracy to the number of estimates, the relative performances of the  
716 methods in terms of accuracy change according to the number of estimates. In other terms, as is the case for the LCSF and  
717 CSF methods in Fig. 8, one method may outperform another method depending on the number of estimates we consider.  
718 Indeed, below 1000 estimates, the CSF method is characterised by a better precision than the LCSF method for all the  
719 statistical indicators and in particular for the 95<sup>th</sup> percentile of the deviation distribution. The best performance of the CSF  
720 methods in terms of precision is then reached for ~400 estimates where the median of the deviations is ~25% compared to  
721 ~29% for the LCSF method. But, if the number of estimates increases beyond 1000, the LCSF method starts outperforming  
722 the CSF method with respect to the 95<sup>th</sup> percentile and when estimates are not filtered by their QI (right ends of the curves of  
723 Fig. 8), it totally outperforms the CSF method not only in terms of precision but also in terms of number of estimates: if all  
724 estimates are considered, the LCSF/CSF method generates 2722/2028 estimates whose deviations from the truth are  
725 characterised by an IQR of 17%–56%/17%–75%. Furthermore, the LCSF method discards outliers much more efficiently  
726 than the CSF method insofar as the 95<sup>th</sup> percentile of the deviation distribution is much lower for the former (118%) than for  
727 the latter method (341%).

728 Selecting one method over another involves making a trade-off between precision and the number of estimates obtained.  
729 Taking the example from Fig. 8, if the primary objective of an application is to obtain as many estimates as possible, the  
730 LCSF method would be the preferred choice, as it can provide 2722 estimates with an IQR of the deviations ranging from  
731 17% to 56%. On the contrary, if the main priority is to obtain estimates with the highest precision, the CSF method would be  
732 more suitable, providing approximately 400 estimates with an IQR of the deviations ranging from 11% to 45%. The trade-off  
733 between accuracy and number of estimates in the choice of method is even more accentuated in the case where inversions  
734 are made with ERA5, as the use of this wind product increases the accuracy of the CSF method through bias compensation  
735 (Sect. 3.4): in this case, using the CSF method, a maximum precision can be obtained, with an IQR equal to 11%–42%, for  
736 650 estimates. If, on the other hand, the LCSF method is used, a maximum number of estimates, 2670, can be obtained with  
737 an IQR of 18%–55% (Fig. A6).

738 The difficulty in achieving the best possible precision for a given method lies in determining an appropriate QI for their  
739 estimates. Here, we adopted a relatively simple approach by defining high-quality estimates as those with relative or absolute  
740 errors below the 10<sup>th</sup> percentile of the distribution relative to all the uncertainties of the estimates. However, as seen in the  
741 curves of Fig. 8, highest precision may not be achieved at this value but at a higher one as in the examples of the IME and  
742 CSF method. This is because misleading estimates, such as those resulting from the overlap of plumes from two sources, can  
743 be characterised by very small uncertainties but at the same time by important deviations from the truth, and their impact on  
744 the results becomes significant when the number of estimates gets relatively small. More generally, the QIs defined in this  
745 study reflect the actual uncertainties in the estimates more or less well and the definition of a more reliable QI that ensures  
746 increased accuracy with higher values of the indexes and deliver the maximum achievable precisions for all of the methods  
747 is beyond the scope of this study, as it likely requires extensive studies in order to provide a common and an accurate  
748 characterization of the total uncertainties in the estimates for all the inversion methods. Finally, we will note that all the

749 qualitative insights stated above about the relationships between accuracy and number of estimates are also valid when  
750 considering inversions using cloud-filtered data and ERA5 winds (Fig. A7).

### 751 **5.3 Single methods vs ensemble approaches**

752 In this study, we create ensemble approaches by averaging the single-image estimates – for the same source and from the  
753 same individual image – produced by different inversion methods. The aim is to obtain more robust and reliable predictions  
754 if individual biases and errors associated with each approach compensate each other. We want thus to analyse whether an  
755 ensemble method, although more expensive from a computational point of view, would perform quantitatively better than a  
756 single method among CSF, GP and LCSF; these methods clearly outperforming the IME method in terms of accuracy and  
757 number of estimates.

758 Four sets of ensemble approaches are considered: the first one integrates the CSF, GP and LCSF inversion methods, and  
759 the remaining three ensemble approaches integrate pairs of methods (CSF & GP, CSF & LCSF and GP & LCSF). Moreover,  
760 in order to assess the impact of the QIs of the different inversion methods on the performance of the ensemble methods,  
761 results are analysed by considering 1) all the estimates and 2) only the best estimates produced by each method. As results  
762 are assessed for the inversions using ERA5 winds and cloud-filtered data which provide a relatively small number of  
763 estimates, we consider the best estimates as the estimates whose relative/absolute errors are below the 25<sup>th</sup> percentile of their  
764 respective error distribution.

765 The ensemble approaches do not provide clear improvements in terms of estimate accuracy over the individual methods  
766 from which they are derived (Fig. 9), with the exception of the important number of outliers produced by the CSF method  
767 when estimates are not filtered: the 95<sup>th</sup> percentile of the deviation distribution is equal to 286% for the CSF method only,  
768 while it decreases to 160% for the ensemble approach gathering the CSF, GP and LCSF methods. On the other hand, the  
769 skewness of the CSF distribution of deviations lead to an increase of the 95<sup>th</sup> percentile of the deviations of the ensemble  
770 approaches compared to the 95<sup>th</sup> percentiles of the LCSF and GP methods. Otherwise, the IQR of the deviations are similar  
771 for all the ensemble and individual approaches and roughly ranges from 15% to 65% when estimates are not selected based  
772 on their uncertainty and from 15% to 60% when the best estimates are selected. Therefore, errors and biases in the estimates  
773 produced by a given method are generally not compensated by the estimates of other inversion methods which suggest that  
774 in general, for the same images and sources, the estimates produced by other inversion methods may also present larger  
775 errors or similar biases.

776 The great benefit of using ensemble approaches lies in the significant increase in the number of estimates, which is a  
777 crucial issue in the real world when the amount of satellite data is strongly limited by the cloud cover. The ensemble  
778 approach gathering the CSF, GP and LCSF methods can supply a maximum of 412 estimates over the year analysed in this  
779 study, representing a 30% increase compared to the LCSF method which is the individual method that supplies the most  
780 estimates (318). This result indicates that the CSF, GP and LCSF methods can provide estimates from different images, i.e. if  
781 one method does not provide an estimate from a given image, another method from the ensemble may, conversely, provide



782 one (Fig. A8). This allows the ensemble method to produce a maximum number of estimates (412) that is close to the  
783 number of usable satellite images (~500). When only best estimates are considered, the ensemble approach generates more  
784 than twice as many values compared to the LCSF method (195 vs 80) whereas the other ensemble approaches (CSF & GP,  
785 CSF & LCSF and GP & LCSF) only provide about 140 estimates.

786 While combining the estimates generated by the CSF, GP and LCSF methods seems to be the optimal choice for an  
787 ensemble approach providing the largest number of predictions, the computational cost of using these methods together may  
788 not outweigh the benefits in terms of number of estimates compared to using a single method. For example, in the most  
789 realistic scenario of inversions conducted with cloud-filtered data and ERA5 winds, the computational time required for the  
790 CSF-GP-LCSF ensemble method is more than three times that of the LCSF method alone (see Sect. 2.1) whereas the overall  
791 precision of the LCSF method is better and the increase in the number of estimates is only 30% when using the ensemble  
792 approach. Therefore, if the performance of computer systems remains an important factor to take into account, one would  
793 prefer to use the LCSF method, which is the fastest method of this study, instead of using an ensemble approach.

794 In order to investigate the benefit of using ensemble approaches for the estimation of annual emissions, we use the same  
795 three individual methods that produce much better results than the IME and Div methods (see Sect. 4.1), but we consider  
796 different definitions of the annual estimates depending on the inversion method: annual estimates are arithmetic means of  
797 image estimates for the LCSF and the GP methods whereas they are weighted means for the CSF method. This choice  
798 corresponds to the best performance at the annual scale that has been found in this study for each method (Sect. 4.1.)  
799 Besides, no selection of the estimates was performed to compute the annual estimates although the quality of the estimates is  
800 integrated within the annual estimates of the CSF method which are averages weighted by the errors in the estimates. Among  
801 the ensemble methods considered here, only the approach gathering the CSF and GP methods yields better results than the  
802 best individual method composing it for most of benchmarking scenarios (Fig. A9). For example, when inversions are  
803 performed with cloud-filtered data and SMARTCARB winds, the CSF, GP and their ensemble approach are characterised by  
804 relative RMSE equal to 18%, 20% and 16%, respectively. The benefit of using ensemble methods for estimating annual  
805 estimates is thus questionable, especially considering that the gain in accuracy, if any, is very small compared to the  
806 individual methods which, depending on the inversion scenario, produce the more accurate annual estimates. This is due to  
807 the fact that the inversion methods generate annual estimates that are generally biased in the same way: emissions of strong  
808 sources are generally underestimated while emissions of weak sources are generally overestimated (see median values in  
809 Fig. 6).

## 810 **6 Conclusions**

811 In this paper, we tested and benchmarked several lightweight data-driven inversion methods for estimating local (city and  
812 power plant) emissions from XCO<sub>2</sub> and NO<sub>2</sub> satellite images. The five methods that have been studied are the Integrated  
813 Mass Enhancement (IME), the Cross-Sectional Flux (CSF), the Gaussian Plume (GP), the Light Cross-Sectional Flux

814 (LSCF) and the Divergence (Div); this last method generating only annual estimates. In a domain centred over the city of  
815 Berlin, which extends about 750 km in the east-west and 650 km in the south-north direction, inversions were performed  
816 with almost one year of synthetic SMARTCARB XCO<sub>2</sub> and tropospheric column NO<sub>2</sub> satellite observations with similar  
817 characteristics as the upcoming CO2M mission. The ability of the inversion methods to estimate emissions has been assessed  
818 by comparing the deviations of estimates from the corresponding “true” values used in the simulations, for 16 sources  
819 including the city of Berlin and 15 power plants. To get a complete overview of performance, several benchmarking  
820 scenarios were considered in order to analyse the benefit of using auxiliary NO<sub>2</sub> data or the impacts of the cloud cover in the  
821 data or of uncertainties in the wind data.

822 In terms of quantifying emissions from single satellite images, the implementations of the CSF, GP and LCSF methods  
823 used in this study outperform that of the IME method. Furthermore, we have demonstrated that the performance in terms of  
824 accuracy and number of estimates varies, to a greater or a lesser extent depending on the method, with the selection of the  
825 estimates based on their relative or absolute uncertainty. The overall accuracies of the IME and CSF methods are  
826 significantly enhanced when a strict screening for high quality estimates is applied but at the cost of an important decrease in  
827 the number of estimates. The GP and LCSF methods, on the other hand, perform more robustly showing only a variation in  
828 their global precisions with increasing quality screening. This behaviour points out the need for these methods of a better  
829 characterization of the uncertainties in the estimates. When estimates are filtered, the CSF method yields the best results in  
830 terms of accuracy while, when estimates are not filtered, the LCSF method provides the highest number of estimations with  
831 a slight decrease in accuracy. Overall, the CSF, GP and LCSF methods show similar accuracies for all the benchmarking  
832 scenarios and when the less reliable estimates of the CSF method are removed: most of IQRs of the absolute deviations  
833 range from 15% to 60% with an average median around 35%. Moreover, for the most realistic benchmarking scenario, i.e.  
834 for the inversions using cloud-filtered NO<sub>2</sub> & CO<sub>2</sub> data and ERA5 winds, the IME, CSF, GP and LCSF methods generate on  
835 average 6 (IME), 18 (CSF), 17 (GP) and 20 (LCSF) estimates per source and per year with great differences between sources  
836 (See Sect. 3.3), which is equivalent to a maximum number of estimates equal to 96 (IME), 295 (CSF), 274 (GP) and 318  
837 (LCSF) for all 16 sources. These figures are significantly lower than the number of usable images (~500) that can provide a  
838 hypothetical constellation of 3 satellites as analysed here; this suggests that methodological improvements could increase the  
839 number of estimates.

840 The accuracy of the CSF and IME methods was found to depend on the strength of the sources with important errors  
841 when determining low emissions; the GP and LCSF methods, in contrast, show similar performances across different ranges  
842 of emissions. Moreover, the advantage of using co-located NO<sub>2</sub> signal for plume detection and quantification appeared to be  
843 clear for the CSF, IME and GP methods, for which the number of single-image estimates significantly increased, while it  
844 was rather weak for the LCSF method. When a cloud cover mask was taken into account in the data, the number of estimates  
845 significantly decreased for all the inversion methods with an average reduction of 85%; the global precision however hardly  
846 decreased and even improved for the IME method. For all the inversion methods, the sensitivities of the results to wind  
847 uncertainties were surprisingly found to be insignificant when replacing the SMARTCARB winds (used in the simulation)

848 by ERA5 reanalysis winds. Finally, if we do not take computational cost into account, the interest in using ensemble  
849 approaches instead of a single method lies mainly in an increased number of single-image estimates as the availability of  
850 estimates from the different methods complements each other.

851 Part of the effectiveness of the implementations of the cross-sectional flux method may come from the generation of  
852 multiple estimates of cross-sectional fluxes along plumes and the subsequent averaging in order to get an unique emission  
853 estimate for a given source and satellite overpass. Probably, errors in the satellite data or in the simplifying assumptions of  
854 the cross-sectional approaches partly cancel out when averaging. The CSF implementation uses a complex algorithm of  
855 plume detection which makes it possible to use the total detectable plume, probably leading to more accurate estimates than  
856 for the LCSF implementation, which only uses observations near the source. However, the plume detection and the  
857 computation of the curved centreline can fail for weak sources (i.e. short plumes) at the cost of having a large number of  
858 outliers. On the contrary, the LCSF implementation uses a simpler but more robust algorithm that uses the wind vector to  
859 estimate the location of the plume, which likely explains why this method generates more estimates, and without the need of  
860 NO<sub>2</sub> data, compared to the CSF implementation. However, efforts should be made to correct the systematic underestimation  
861 of strong emissions by the LCSF implementation. A way forward can be merging the CSF and LCSF method into a single  
862 algorithm that takes the advantages of both approaches.

863 When compared to other methods, the relative ability of the GP method in estimating emissions probably relies on the  
864 use of a Gaussian function whose optimization determines the emissions while taking into account the entire structure of the  
865 plumes, and calculating effective winds that are consistent with that of the plumes. However, this optimization and thus the  
866 performance of the GP method highly depend on the first-guessed values to be assigned to its parameters (not shown). And,  
867 in this study, the first-guessed values of the emissions are the summer average emissions for each source; this could be a  
868 strong constraint on the estimated values and could lead to an overestimation of the GP performance in this benchmarking  
869 study. Finally, the GP method is computationally expensive due to the heavy plume detection algorithm and to the multi-  
870 parameter optimization required for the Gaussian fitting of the plumes (Table 1).

871 The IME method also integrates information retrieved from the entire structure of the plumes but, contrarily to the GP  
872 method, it does not use this information when computing effective winds. Therefore, these winds may be inconsistent with  
873 the characteristic lengths of plumes used by the IME method to estimate CO<sub>2</sub> emissions (Sect. 2.1.4) and this could explain  
874 the relatively poor performance of the IME method in this study. Varon et al. (2018) probably found that the IME method  
875 was adapted to estimate CH<sub>4</sub> emissions from high-resolution plumes because they inferred a relationship between the  
876 effective winds and the characteristic lengths through LES simulations. Another drawback of the IME method is that it is  
877 very sensitive to missing data as it needs an entire coverage of the plume area by data to efficiently integrate the total mass  
878 enhancement. Other single-image methods (GP, CSF and LCSF) are less sensitive to missing data as they fit functions to the  
879 data and can handle data gaps; this explains why these methods provide a much larger number of estimates when the impact  
880 of cloud cover on the data is considered (see Sect. 3.3).

881 In this study, we chose not to analyze the potential of the divergence method for estimating instant emissions from single  
882 satellite overpasses because of the lack of studies on such an application of this method. As highlighted in the introduction  
883 section, our aim is to compare proven approaches for the local scale estimation of strong sources (such as the application of  
884 the divergence method to time-averages of satellite images). Moreover, the strong spatial variability of the divergence fields  
885 derived from single images suggest that only averaged fields could be processed properly with the version of the divergence  
886 approach which is used here for annual estimates and which relies on the peak-fitting of temporally averaged divergence  
887 fields. However, we have conducted some preliminary analysis on a version of the divergence method which instead  
888 integrates the divergence signal spatially (over disks centered on the sources). The results, documented in appendix A,  
889 demonstrate that with a range of integration radii close to that of the spatial resolution of image, this approach can yield  
890 estimates that would be comparable in terms of accuracy and quantity to that of the best inversion methods of our benchmark  
891 evaluation for single-image based estimates. A better understanding of the behavior of this approach as a function of the  
892 integration radius, and an assessment of the estimation errors are needed to conduct a proper comparison to the other  
893 methods. This deserves further investigations. However, these preliminary results raise optimistic perspectives regarding the  
894 potential of using the divergence method for estimating instant emissions from single-overpass images.

895 For estimating annual emissions, the CSF, GP and LCSF methods outperform the Div and IME methods when annual  
896 estimates are computed as error-weighted means of single-image estimates for the CSF method and as arithmetic means of  
897 these estimates for the GP and LCSF methods. Across the different benchmarking scenarios, the GP method shows better  
898 precisions in its annual estimates because its single-image estimates have similar absolute deviations from the truth but are  
899 less affected by biases compared to the CSF and LCSF methods (see Fig. 3). However, despite biases, errors in the single-  
900 image estimates provided by the CSF, GP and LCSF methods likely compensate when averaging and these methods also  
901 generate annual estimates with a better precision than for their single-image estimates. In the most realistic benchmarking  
902 scenario – where inversions use cloud-filtered XCO<sub>2</sub> & NO<sub>2</sub> data and ERA5 winds and where performances are the lowest  
903 compared to other scenarios – the relative RMSE for the annual emissions of the 16 sources is 20% (GP), 27% (CSF), 31%  
904 (LCSF), 55% (IME) and 79% (Div). The relatively weak performance of the Div method could be explained by the fact that  
905 this method was originally developed for the estimation of NO<sub>x</sub> emissions and the fields of this chemical species are  
906 generally characterised by stronger divergence peaks than for CO<sub>2</sub> fields. However, its performance could be improved by  
907 selecting and averaging images that are characterized by favourable conditions such as strong signals or wind speeds  
908 important enough to guarantee the predominance of advective processes in the atmospheric transport. The performances of  
909 ensemble approaches gathering several inversion methods in terms of annual estimations is not better, and in some cases  
910 even worse, than the individual methods. Finally, none of the methods were able to correctly reproduce the monthly seasonal  
911 cycle of the emissions when data underwent a cloud-filtering, i.e. when data were not available for some months, which  
912 points out the need for an extensive temporal coverage of the observations when aiming to capture the monthly variability in  
913 emissions.

914 In addition to the technical improvements that could be made on the algorithms of the methods, further developments  
915 could extend this study such as the integration of new data streams for estimating CO<sub>2</sub> emissions such as satellite data of  
916 other co-emitted gases than NO<sub>2</sub>, e.g. CO data provided by the TROPOMI instrument. A companion paper (Hakkarainen et  
917 al., 2024) analyses the ability of the inversion methods in determining NO<sub>x</sub> emissions, from synthetic and TROPOMI NO<sub>2</sub>  
918 satellite data for the Matimba and Medupi power plants in South-Africa. The NO<sub>2</sub> synthetic data are extracted from the high-  
919 resolution MicroHH Large Eddy Simulations (LES) (Van Heerwaarden et al., 2017) and used in particular to study the  
920 nitrogen dioxide to nitrogen oxide scaling factors that are required for satellite-based estimations of NO<sub>x</sub> emissions.  
921 Moreover, the capacity of the inversion methods to estimate city emissions has been analysed in this study on the single  
922 example of the city of Berlin and, as most of the methods have provided correct estimates for its emissions, it would be  
923 interesting to expand this study to other cities and other local sources. Finally, this benchmarking study has not integrated the  
924 new and promising type of inversion methods that are the methods derived from deep learning techniques (e.g. Lary et al.,  
925 2016). After a potentially complex training phase, deep-learning methods could quickly process large amounts of data and  
926 provide estimations with similar or better accuracy than the methods studied here (Dumont le Brazidec et al., 2023). They  
927 could also complement these methods by allowing a fine differentiation of the plumes compared to the background with  
928 advanced image segmentation techniques.

929 The aim of this study is to contribute to the development of the CO<sub>2</sub> Monitoring and Verification Support system that  
930 will use the upcoming CO2M satellite data. And, although this benchmarking study has been performed with synthetic  
931 observations, the methods studied here can be easily adapted to the analysis of real satellite observations and to deal with  
932 sources of unknown location as demonstrated in Hakkarainen et al. (2024).

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## 937 **Appendix A: Potential of the divergence approach to estimate local CO<sub>2</sub> emissions from single-overpass satellite** 938 **images of XCO<sub>2</sub> and NO<sub>2</sub>**

939 In this study, the performance of the divergence approach to estimate local CO<sub>2</sub> emissions from XCO<sub>2</sub> and NO<sub>2</sub> synthetic  
940 satellite images is assessed with a standard version of this approach (e.g., Beirle et al., 2021; Hakkarainen et al., 2022),  
941 which provides temporally averaged estimates. Results concerning the divergence approach are thus analyzed in the main  
942 part of this paper in terms of annual means. However, following the suggestions of a reviewer (S. Beirle), we also tested the  
943 potential of this method to estimate instant emissions using single-overpass images. For this purpose, we have used two  
944 versions of the divergence approach that have been modified for single image geometry as in Beirle et al. (2023).

945 For both versions, the computation of the divergence fields is performed by only considering the “advective” term  
946 ( $10^6 * M_{air} * U * \nabla(VCD)$ ) of the full expression of the horizontal flux divergence ( $\nabla(10^6 M_{air} * U * VCD)$ ) where  $M_{air}$  is  
947 the dry air mass,  $U$  is the wind vector and  $VCD$  is the vertical column density in parts per million. Such reformulation of the  
948 divergence method that does not compute the divergence of the wind term was also used by Beirle et al. (2023) for  $NO_2$ . The  
949 advantage of this reformulation for  $CO_2$  is that the background (e.g., a constant offset of 400 ppm) is implicitly removed.

950 These versions of the divergence approach differ from each other in their way of computing emissions from the  
951 divergence maps associated with single-overpass images: the first version integrates the divergence fields on disks centered  
952 on the sources (Figure A10). And, to mitigate the impact of the uncertainties in the observations, the emission estimate for a  
953 given satellite overpass and source can be computed as the average of the estimates when integrating the divergence signal  
954 on disks of different radii. This version of the divergence approach will be referred to hereinafter as the *integral* divergence  
955 method. The second version proceeds in a similar way to the one used in the main part of the article and fits a 2-D Gaussian  
956 function to the divergence maps in order to retrieve source emissions (e.g. Beirle et al. 2020). The modified peak fitting  
957 model is similar to the original but with a reduced number of estimated parameters. Namely, the parameters related to the  
958 background and to the location correction are removed from the model parameters. This version of the divergence approach  
959 will be referred to hereinafter as the *peak-fitting* divergence method.

960 For both versions, potential peaks are detected by using  $NO_2$  fields which are integrated over disks of 6 km radius  
961 centered on the sources. If the integral of the divergence map on the disk is larger than the integral on the area outside the  
962 disk, then the enhancement, related to a given source and for a given satellite overpass, is considered strong enough and the  
963 emission estimation can be carried out. Many sources in the SMARTCARB dataset are weak and enhancements may be  
964 barely visible which causes challenges for both versions.

965 To evaluate the potential of these two versions of the divergence approach, we use the SMARTCARB dataset described  
966 in section 2.2. which provides about 3000 images to determine the emissions of the 16 local sources that are considered in  
967 this study (if we take into account the cloud cover, only 500 images remain usable). Furthermore, we consider two  
968 benchmark scenarios (see table 2 and section 2.3) where inversions are performed using  $CO_2$  and  $NO_2$  data with  
969 SMARTCARB winds. In one case, we use cloud-free data, while in the other, cloud-filtered data.

970 The analysis of the deviations from the truth of the instant estimates shows that the integral divergence approach is  
971 strongly sensitive to the radius of the integration disks (Fig. A11). No clear trend appears except that errors increase sharply  
972 for a radius greater than 10 km, with a significant presence of outliers. Below this value, the absolute relative deviations  
973 (bottom panel of Fig. A11) can increase or decrease depending on the value of the radius. Furthermore, the integral  
974 divergence approach can underestimate or overestimate emissions depending if the radius is lower or greater than ~4 km. A  
975 possible explanation for this behavior could be that the impacts of the two main sources of errors in the divergence method  
976 — namely, the uncertainties in the observations and the influence of additional but unwanted sources on the background of  
977 the divergence fields — evolve in opposite directions as the integration radius increases. The impact of the uncertainties is  
978 mitigated when the area of the integration disk increases because errors have more probability to cancel out. Conversely, the

979 impact of neighboring sources on the background of the divergence field intensifies as the integration radius increases,  
980 because the likelihood of capturing features in the divergence maps that are not directly related to the emissions of the  
981 targeted sources grows. This impact consistently introduces a positive bias in the estimates (as we capture more sources) and  
982 is likely more important than the one related to the uncertainties as performance overall degrades when the integration radius  
983 increases.

984 The peak-fitting divergence method is characterized by a poor performance compared to the integral divergence method  
985 for the ensemble of integration radii that we have considered here (Fig. A11). The estimation of small emitting sources may  
986 be more difficult for the peak-fitting version as the fit of the 2-D Gaussian function to the data associated to these sources  
987 often fails and does not provide optimal and reliable parameter combinations, yielding poor and often overestimated  
988 emission estimations. Therefore, even though the peak-fitting divergence method is generally more efficient at the annual  
989 scale, these results suggest that it is not the case when estimating instant emissions from single overpass images.

990 The configuration of the integral divergence method which averages estimates across the integration radii of 2, 3 and 4  
991 km shows the best performance amongst the configurations that we have tested. Probably, the impacts of the data  
992 uncertainties and the background are well balanced for this range of radii and the fact of averaging estimates across three  
993 different radii further reduces the influence of the data uncertainties on the results. When compared to other inversion  
994 methods analyzed in this study, the performance of this configuration of the integral divergence method is similar to that of  
995 the best inversion methods (Fig. A12). For the benchmarking scenario considering cloud-free data, its relative absolute  
996 deviations are for example characterized by a median value of ~38% and Interquartile Range (IQR) of [~19% – ~64%]  
997 which are comparable to deviations associated to the Light Cross-Sectional Flux (LCSF) method which have a median value  
998 of ~32 % and an IQR of [~15 %~56 %]. Note that the integral divergence method generates fewer estimates (2174)  
999 compared to the LCSF method (2722), but more than the Gaussian Plume (GP) method (1776).

1000 These preliminary results regarding the potential of the integral divergence method for estimating local CO<sub>2</sub> emissions  
1001 from single-overpass images of XCO<sub>2</sub> and NO<sub>2</sub> appear promising, especially since this method allows for the detection of  
1002 plumes from unknown sources (Beirle et al., 2021). However, further investigation is required to properly assess factors such  
1003 as the integration radius based on data resolution, and to generalize this method to various types of satellite data.  
1004 Additionally, a thorough quantitative error assessment is essential to evaluate the accuracy of the estimates, enabling the  
1005 classification and selection of estimates, which would enhance the method's overall performance.

1006  
1007 *Code and data availability.* The code repository of the python package *ddeq* is available on Gitlab.com:  
1008 <https://gitlab.com/empa503/remote-sensing/ddeq>. The SMARTCARB dataset is available on Zenodo:  
1009 <https://doi.org/10.5281/zenodo.4048227>.

1010  
1011 *Author contributions.* DS made the diagnostics and led the analysis for the intercomparison of the results from the different  
1012 inversion methods. All co-authors contributed to the decisions for the configuration, diagnostics and analysis of the

1013 intercomparison. DS wrote the manuscript with inputs from all co-authors. DS, GB and FC carried out the analysis specific  
1014 to the LCSF method. JH, II, HL, JN and LA carried out the analysis specific to the Div method. GK developed the original  
1015 ddeq library that has been used as a basis for the application of the different methods. GK provided the SMARTCARB  
1016 dataset used to test the different methods. GK carried out the analysis specific to the IME method. EK carried out the  
1017 analysis specific to the CSF and GP inversion methods. The project was coordinated by JT, DB and GB.

1018

1019 *Competing Interests.* Some authors are members of the editorial board of Atmospheric Measurement Techniques. The  
1020 authors have no other competing interests to declare.

1021

1022 *Acknowledgements.* Most of the work performed in this paper was done in the framework of EU H2020 project CoCO2  
1023 (grant No. 958927). The FMI team would like to thank the Research Council of Finland project 353082. All authors would  
1024 like to thank the ICOS Carbon Portal for providing access to their JupyterLab servers, which were used for code development  
1025 and data sharing. Finally, the authors would like to thank the two reviewers for their insightful comments, and especially S.  
1026 Beirle for his suggestions on the application of the divergence approach for estimating instant emissions.

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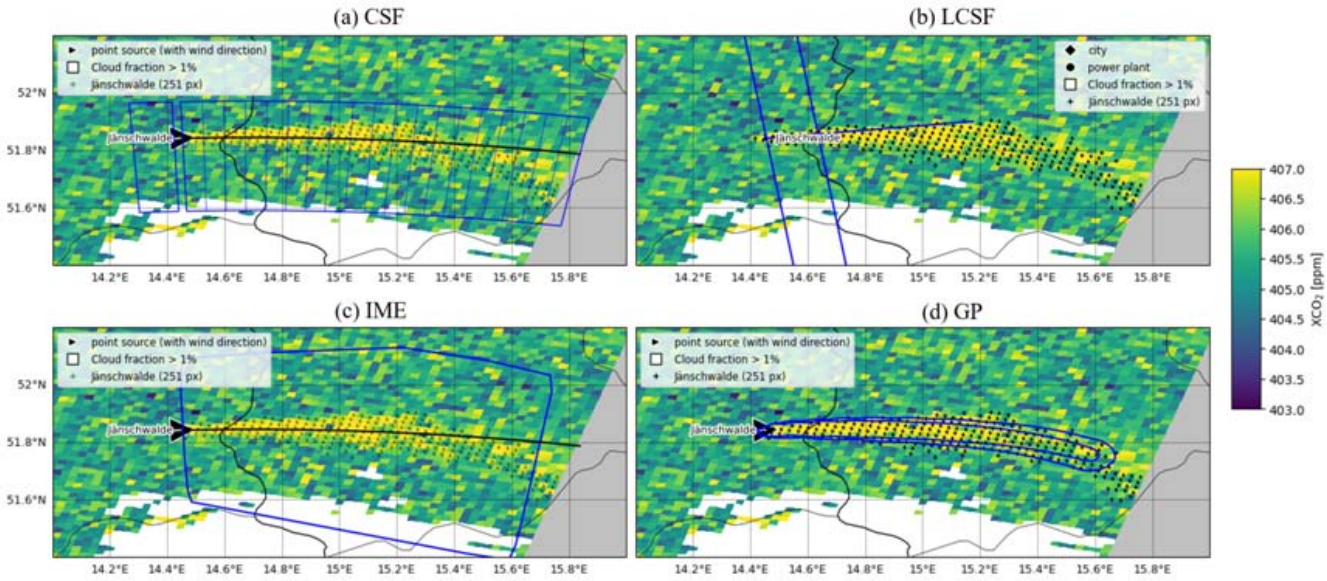
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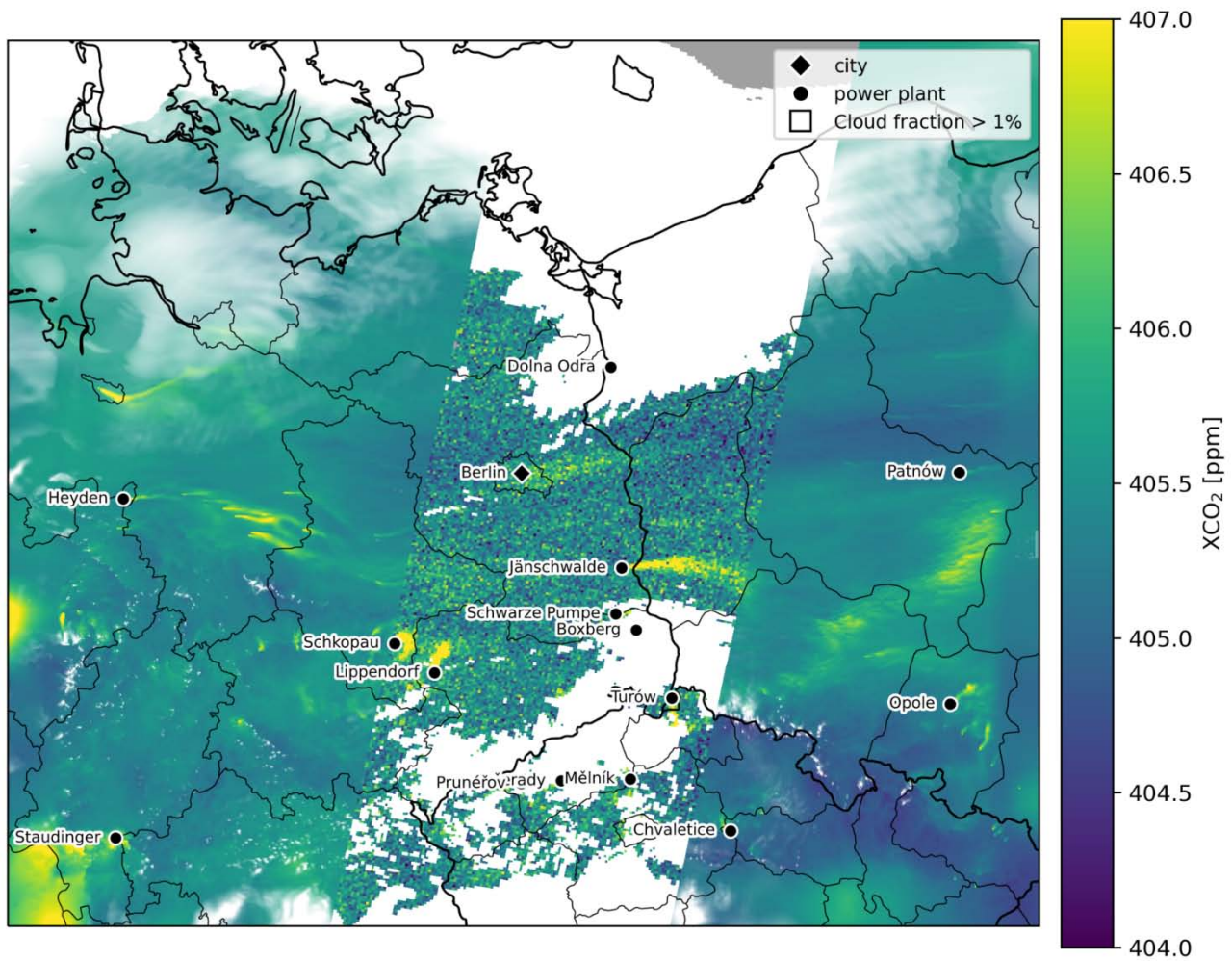
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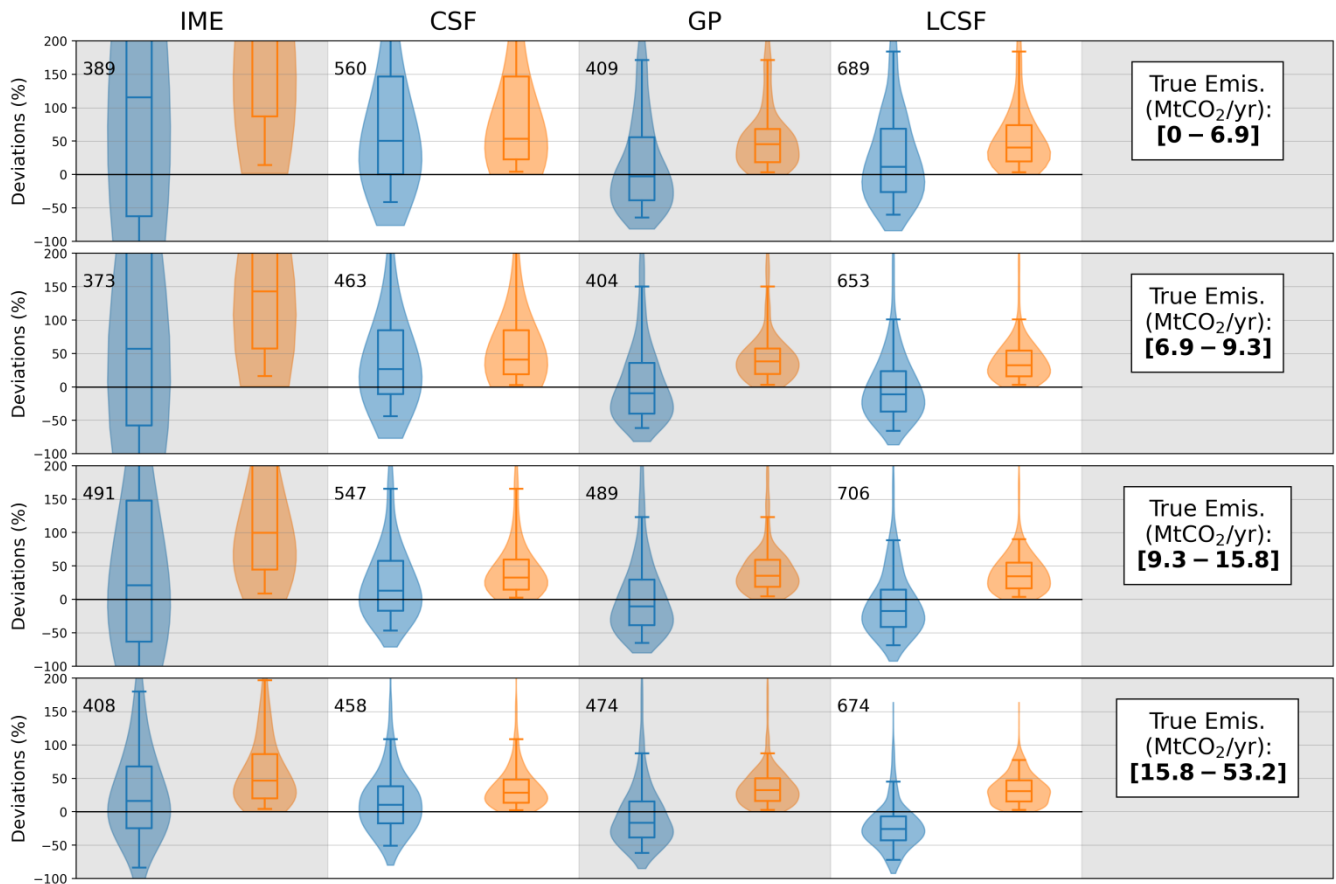
**Figure 1. Illustration of different inversion methods for a plume produced by the Jämschwalde power plant on April 23<sup>rd</sup>, 2015. For all figures, pixels with dots are the selected enhancements representing the plume a) CSF method: the blue boxes depict the areas where the Gaussian fits of the plume cross-sections are made and the black line the centre-line of the plume. b) LCSF method: the blue lines represent the domain where the Gaussian fits of the plume cross-sections are made and the black line the along-wind direction at the source. c) IME method: the blue curve represents the domain on which mass enhancements are integrated. d) GP method: Blue curves depict contour lines of the 2-dimensional Gaussian curve that fits the plume.**





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**Figure 2. Simulations of XCO<sub>2</sub> on 23 April 2015 over the SMARTCARB domain. Synthetic XCO<sub>2</sub> observations over a 250 km wide swath are represented in the centre of the figure for a low noise scenario. Missing XCO<sub>2</sub> observations due to a cloud fraction larger than 1% are shown in white. The 16 emission sources considered in this study are highlighted along with their names**

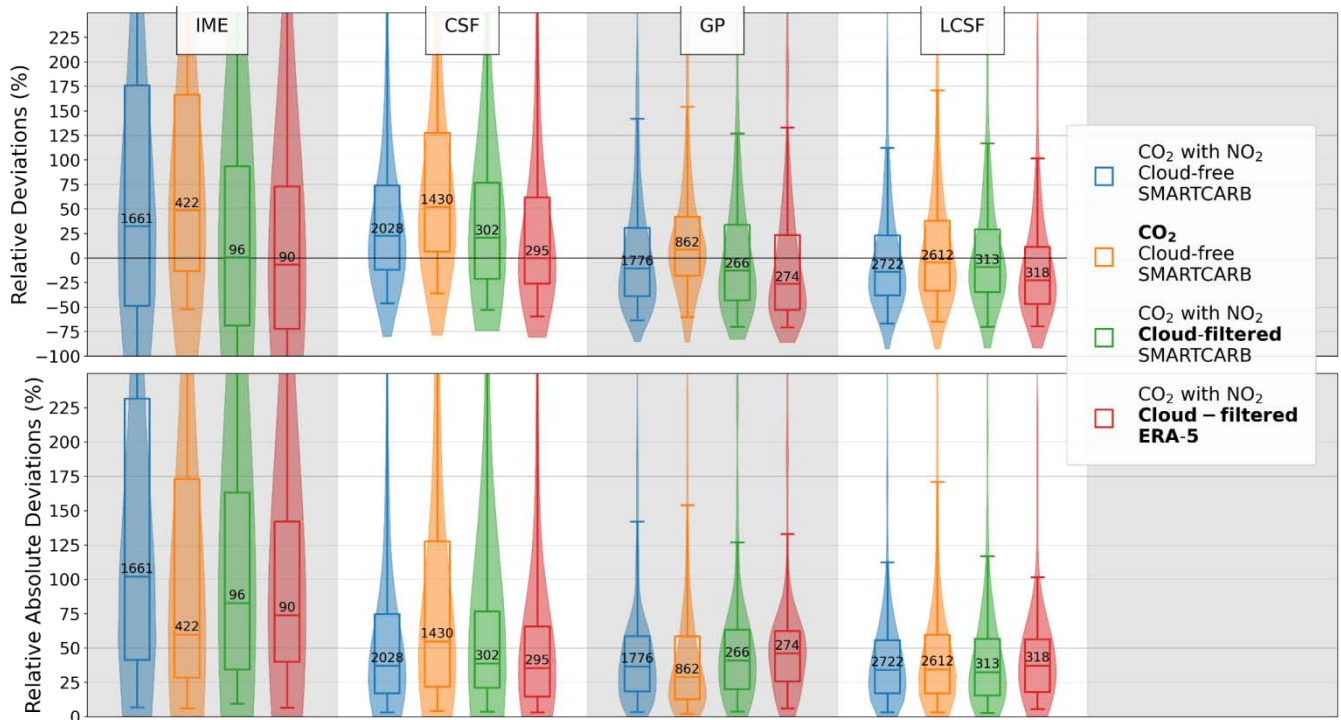


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 1230 **Figure 3. Performance when estimating CO<sub>2</sub> emissions from individual images of the different single-image inversion methods**  
 1231 **(columns) across different ranges of true emissions (rows) using SMARTCARB winds and cloud-free CO<sub>2</sub> and NO<sub>2</sub> data. The**  
 1232 **distributions of relative deviations (in blue) and relative absolute deviations (in orange) are illustrated using violin plots. The inter-**  
 1233 **quartiles are represented by the boxes, while the whiskers indicate the 5<sup>th</sup> and 95<sup>th</sup> percentiles, and medians are the lines inside the**  
 1234 **boxes. The numbers alongside boxes show the numbers of estimates corresponding to true emissions ranges and inversion**  
 1235 **methods.**

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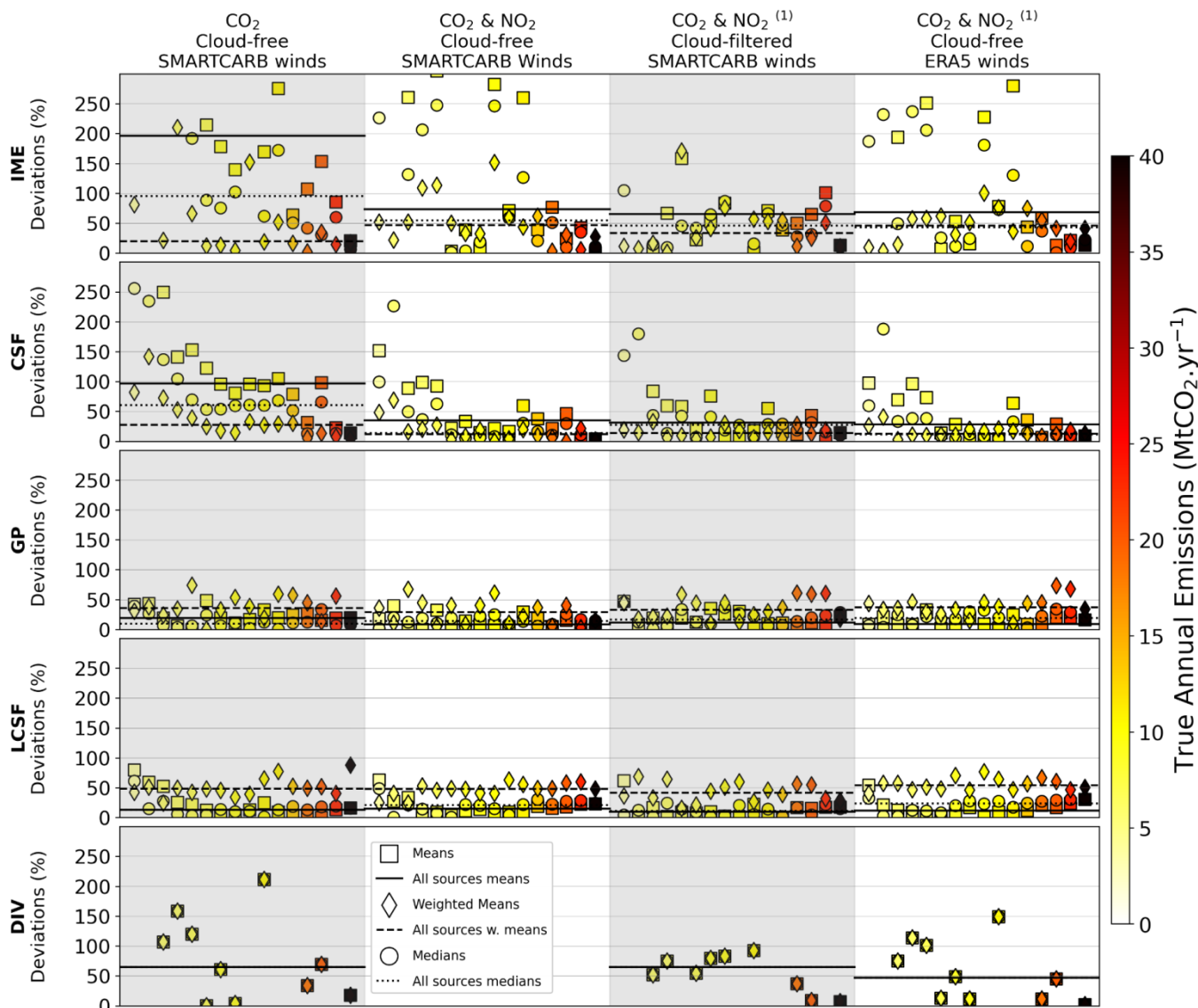
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**Figure 4.** Performances of the inversion methods when estimating emissions from single images for different benchmarking scenarios: cloud-free CO<sub>2</sub> and NO<sub>2</sub> data with SMARTCARB winds (in blue), cloud-free CO<sub>2</sub> data only with SMARTCARB winds (in orange), cloud-filtered CO<sub>2</sub> and NO<sub>2</sub> data with SMARTCARB winds (in green), cloud-filtered CO<sub>2</sub> and NO<sub>2</sub> data with ERA5 winds (in red). **Bold texts in the legend indicate the elements of benchmarking scenarios that differ from those in the ideal benchmarking scenario.** Distributions of the relative deviations (top panel) and relative absolute deviations (bottom panel) are illustrated using violin plots. Boxes are the inter-quartiles of the distributions, the whiskers are the 5<sup>th</sup> and 95<sup>th</sup> percentiles, and the lines within boxes are the medians. Numbers in the inter-quartile boxes are the number of estimates for each benchmarking scenario and inversion method.



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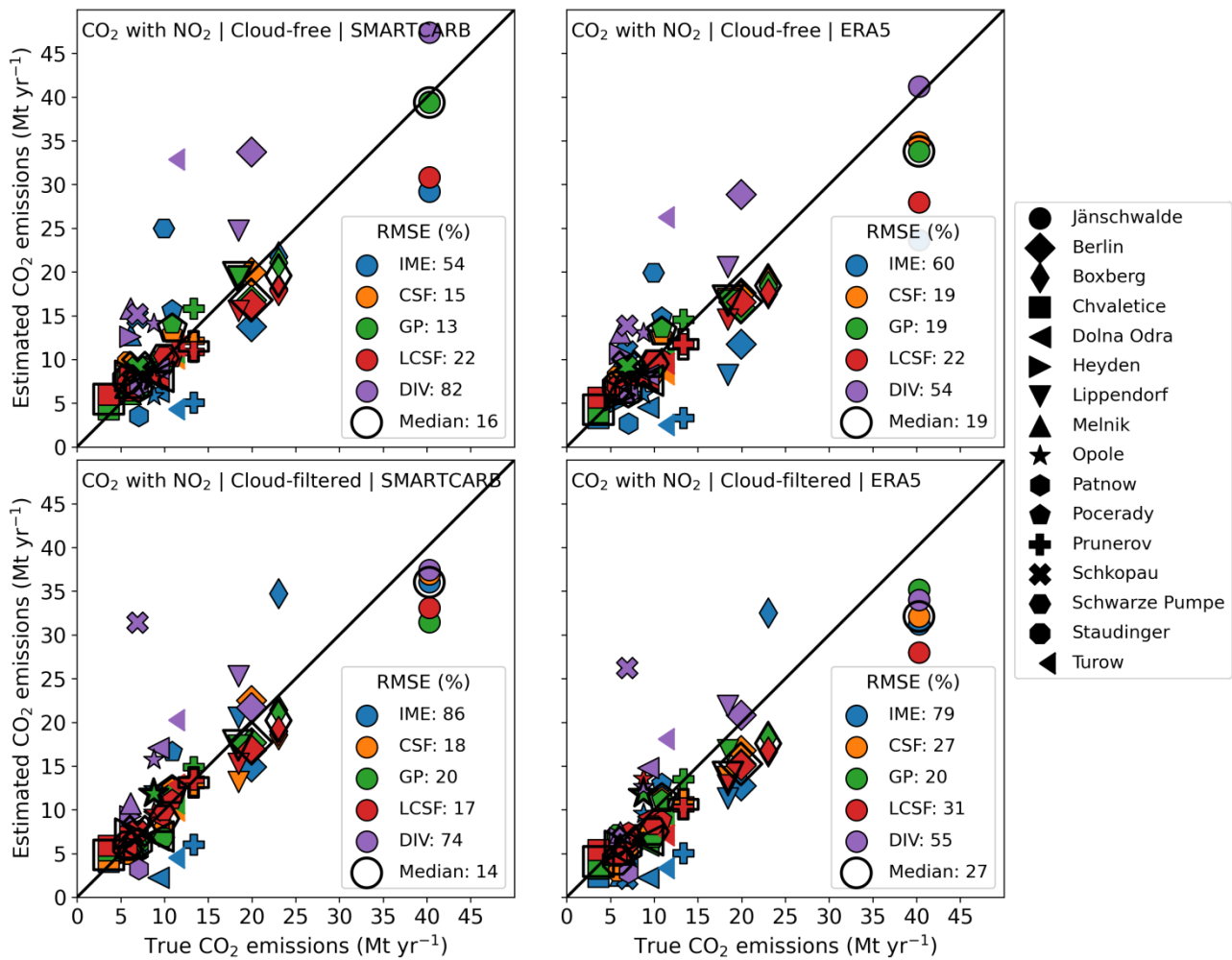
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**Figure 5. Performance of the inversion methods for annual estimates of CO<sub>2</sub> emissions.** The markers represent for a given source the relative absolute deviations from the true annual emissions of the arithmetic means (squares), the weighted means (diamonds) and the medians (circles) of the estimates over a year. The lines represent the median values of the annual estimates over the entire set of sources. The inversions are performed using CO<sub>2</sub> cloud-free data and SMARTCARB winds (1<sup>st</sup> column), using CO<sub>2</sub> and NO<sub>2</sub> cloud-free data and with SMARTCARB winds (2<sup>nd</sup> column), using CO<sub>2</sub> and NO<sub>2</sub> cloud-filtered data and SMARTCARB winds (3<sup>rd</sup> column), and using CO<sub>2</sub> and NO<sub>2</sub> cloud-free data and with ERA5 winds (4<sup>th</sup> column). (1) For the Divergence methods, the inversions of the 3<sup>rd</sup> and 4<sup>th</sup> columns are performed using CO<sub>2</sub> data only. Markers color indicates the true CO<sub>2</sub> annual emissions of the corresponding source.



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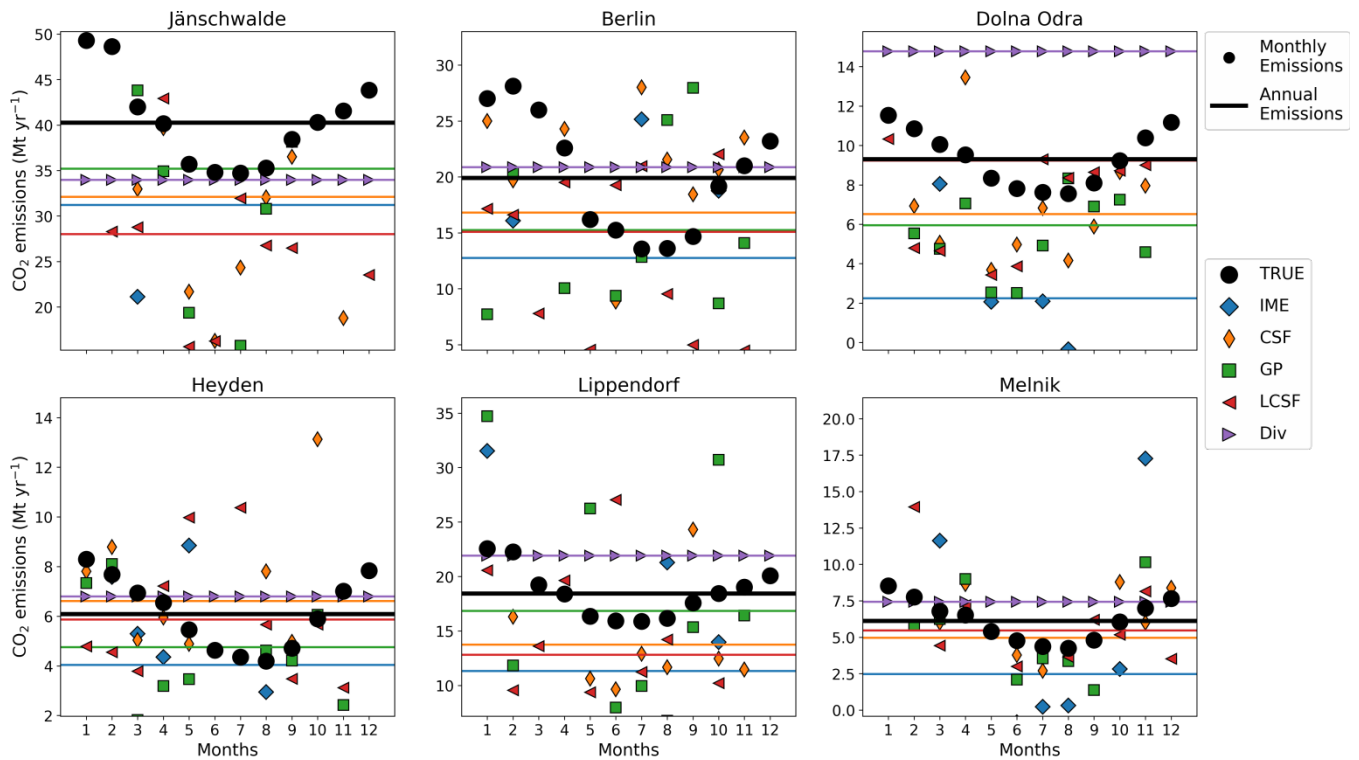
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**Figure 6.** Estimated vs true annual emissions for 4 inversion scenarios (titles of the panels). For the IME and CSF methods, annual estimates are weighted means of the single-image estimates while they are arithmetic means for the GP, LCSF and Divs methods. Each marker represents a given emission source and each color a given inversion method. The unfilled markers represent the median values of all the estimates for each source. The divergence inversion method uses CO<sub>2</sub> data for all the inversion scenarios. The plain line represents the 1:1 line. The bottom-right legends display for each inversion method the relative RMSE which is the RMSE between estimated and true annual emissions divided by the median of true annual CO<sub>2</sub> emissions of all sources (~9.6 Mt yr<sup>-1</sup>).



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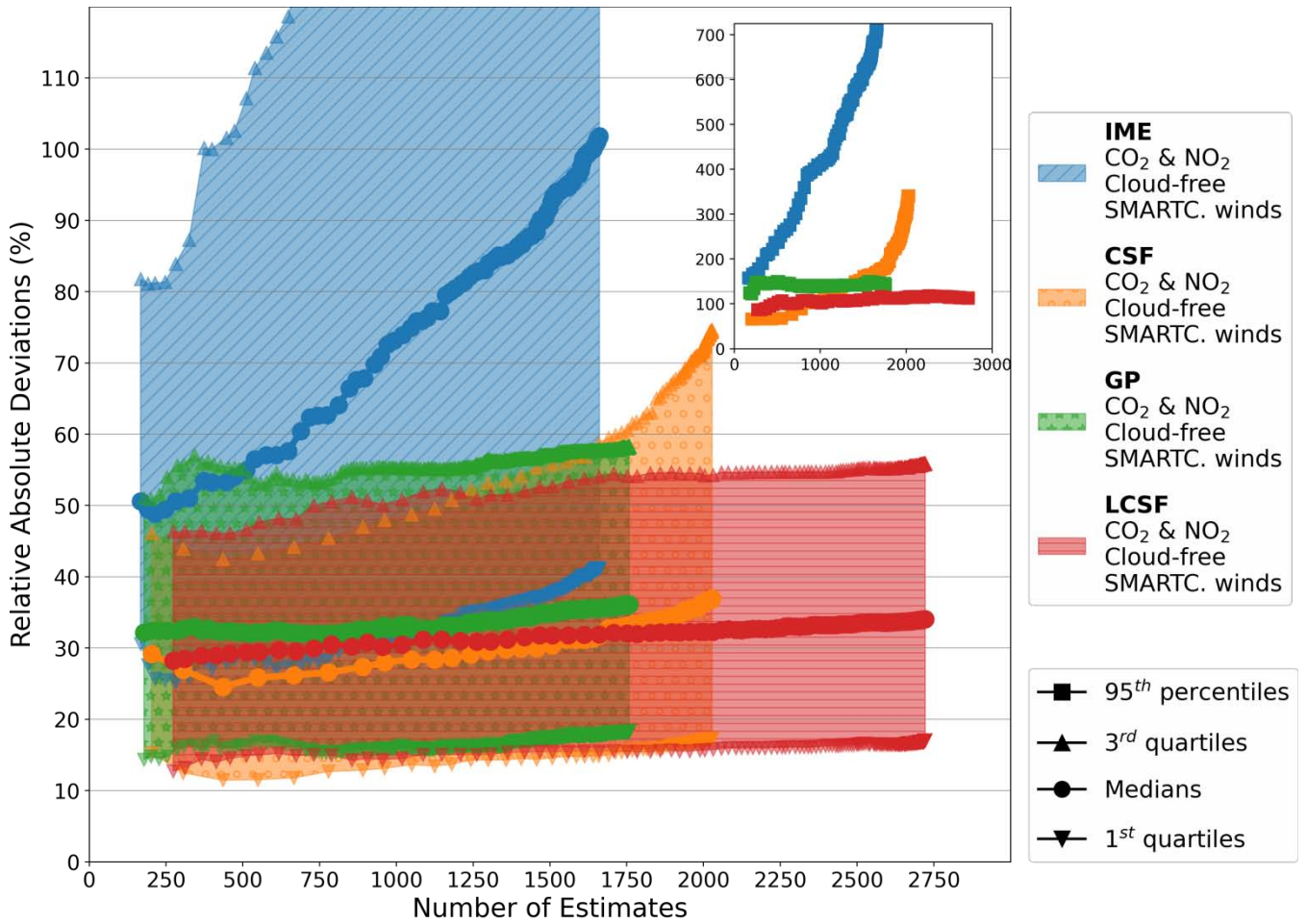
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**Figure 7. Annual and monthly estimates of the true and estimated emissions for different sources and for different inversion methods. Each panel is associated with a given source. Plain lines and markers represent annual averages and monthly averages respectively. Colors and markers are associated with different inversion methods (true emissions are represented by black circles). Annual and monthly estimates for the IME and CSF methods are weighted means of image estimates. Annual and monthly estimates for the GP and LCSF are means of image estimates while for the divergence method, we use the annual estimate also for monthly estimates. All inversion methods use CO<sub>2</sub> and NO<sub>2</sub> cloud-filtered data (CO<sub>2</sub> data only for the Div method) with ERA5 winds.**



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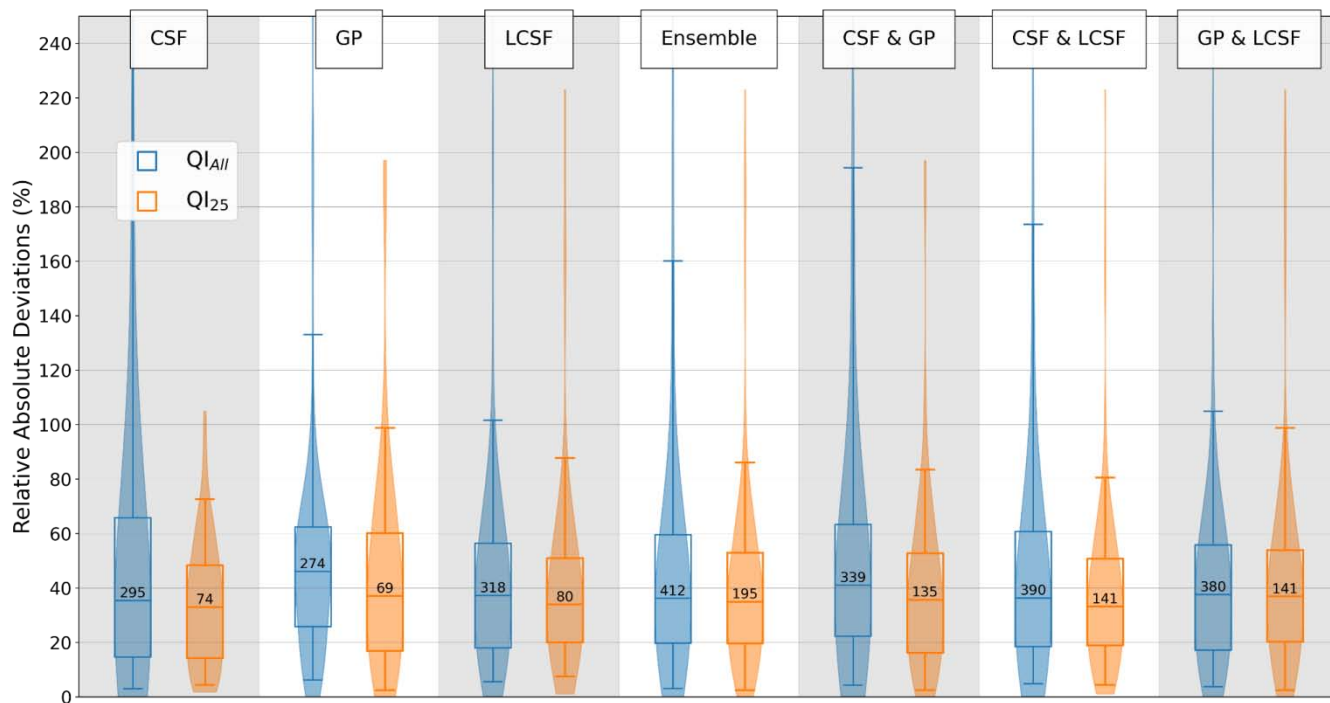
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**Figure 8. Accuracy of inversions vs number of single-image estimates. The inversion methods shown here use CO<sub>2</sub> and NO<sub>2</sub> cloud-free data and SMARTCARB winds. The filled areas represent the inter-quartiles of the distributions of the relative absolute deviations depending on the number of estimates. The 95<sup>th</sup> percentiles of the distributions are represented in the inset. Points belonging to a same curve are associated to different QIs and from left to right along curves, points are associated with a decreasing QI; the points at the left and right ends of the curves are associated with the maximal and minimal QIs respectively.**

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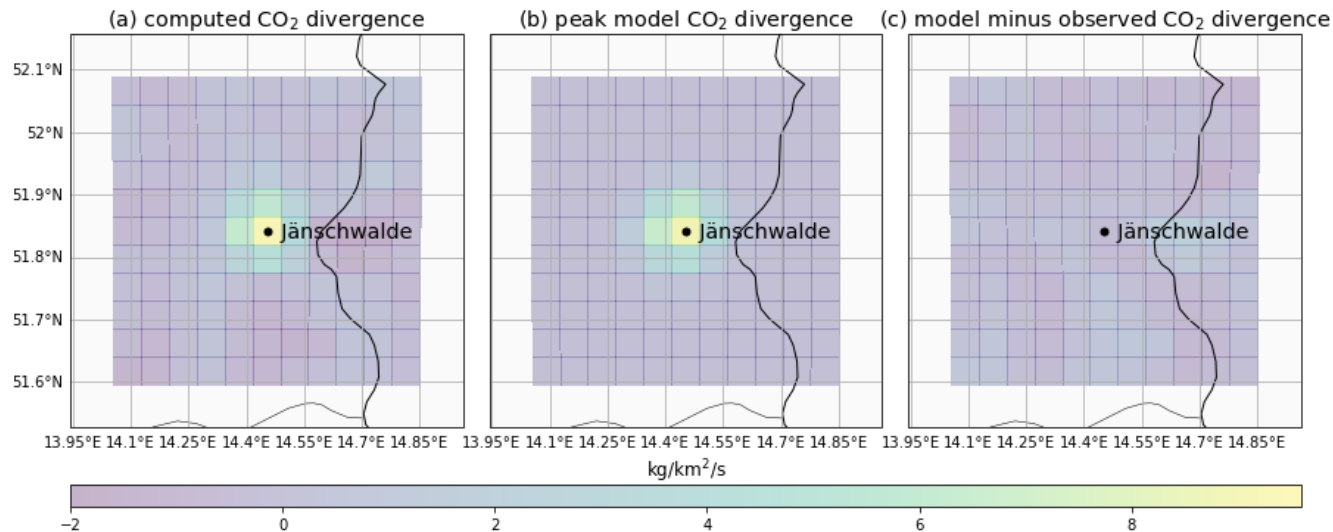
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**Figure 9: Performance of the inversion methods and ensemble approaches for estimating the emissions with cloud-filtered CO<sub>2</sub> & NO<sub>2</sub> data and with ERA5 winds. The distributions of the relative absolute deviations for all the inversion results (in blue) and for the best estimates (in orange) provided by each method (see text) are illustrated using violin plots. Boxes represent the inter-quartiles of the distributions, the whiskers the 5<sup>th</sup> and 95<sup>th</sup> percentiles, and the lines within boxes the medians. Numbers in the inter-quartile boxes are the number of estimates for each benchmarking scenario and inversion method.**



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**Figure A1: Illustration of the divergence method for the Jämschwalde power station in 2015 based on the synthetic SMARTCARB dataset (see text). The figures represent the annual fields of the computed CO<sub>2</sub> divergence (a), the modeled CO<sub>2</sub> divergence (b) and the difference of both quantities (c). Of note that as sink terms are considered negligible for CO<sub>2</sub>, divergence fields are considered equal to the emission fields for CO<sub>2</sub>.**



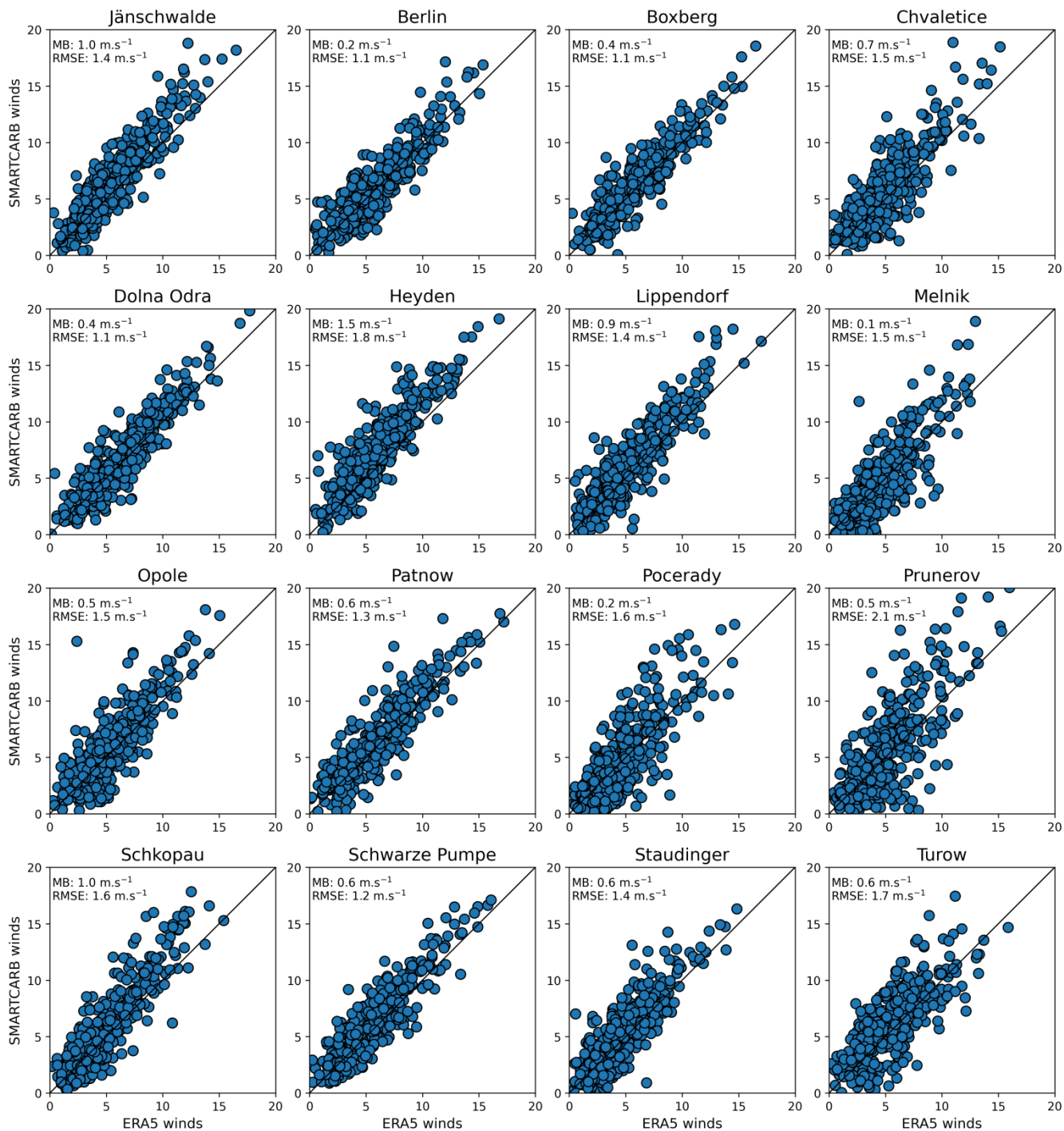
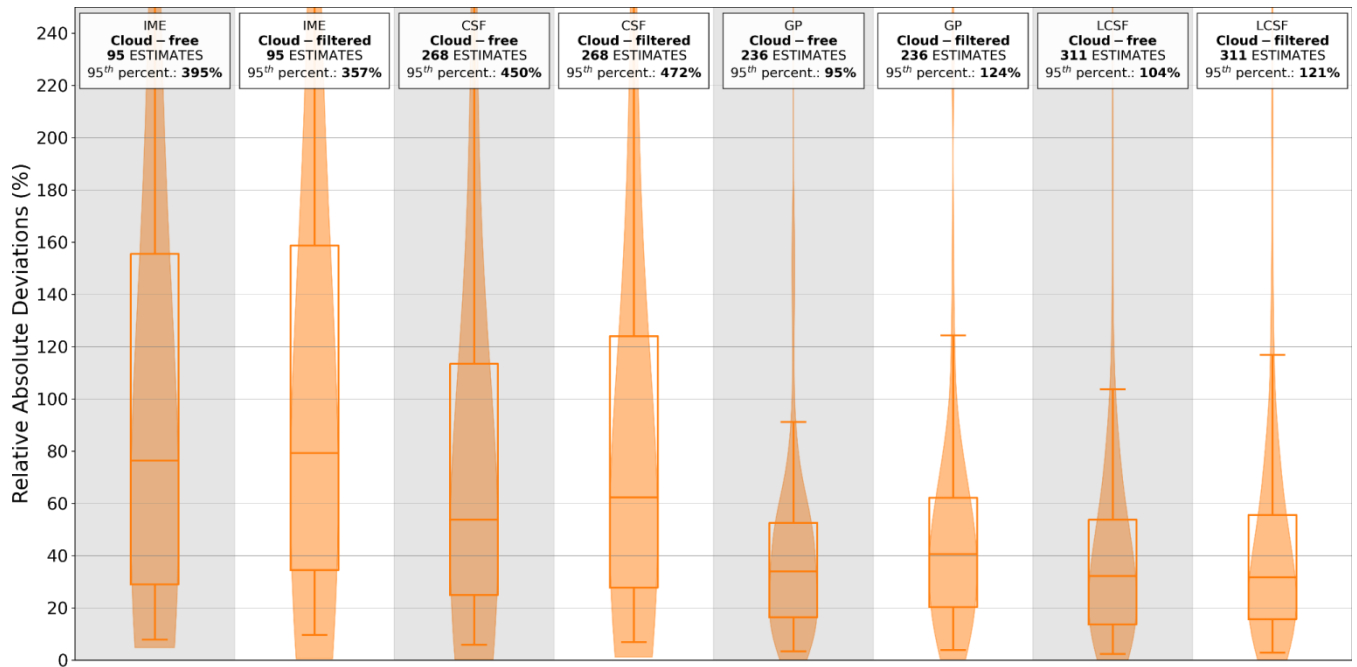


Figure A2: Norms of the ERA5 winds vs norms of the SMARTCARB winds at the sources considered in this study and for all the days of 2015. Black lines represent the 1:1 agreement line. Mean biases of the SMARTCARB norms minus the ERA5 norms and RMSEs are noted at the top left of the figures.

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**Figure A3: Performance of the inversion methods when using data with or without clouds for the emissions estimated from the same images. The inversion methods use CO<sub>2</sub> and NO<sub>2</sub> data and SMARTCARB winds. The boxes represent the inter-quartiles of the distributions of the absolute relative deviations, the whiskers the 5<sup>th</sup> and 95<sup>th</sup> percentiles, and the lines within boxes the medians.**



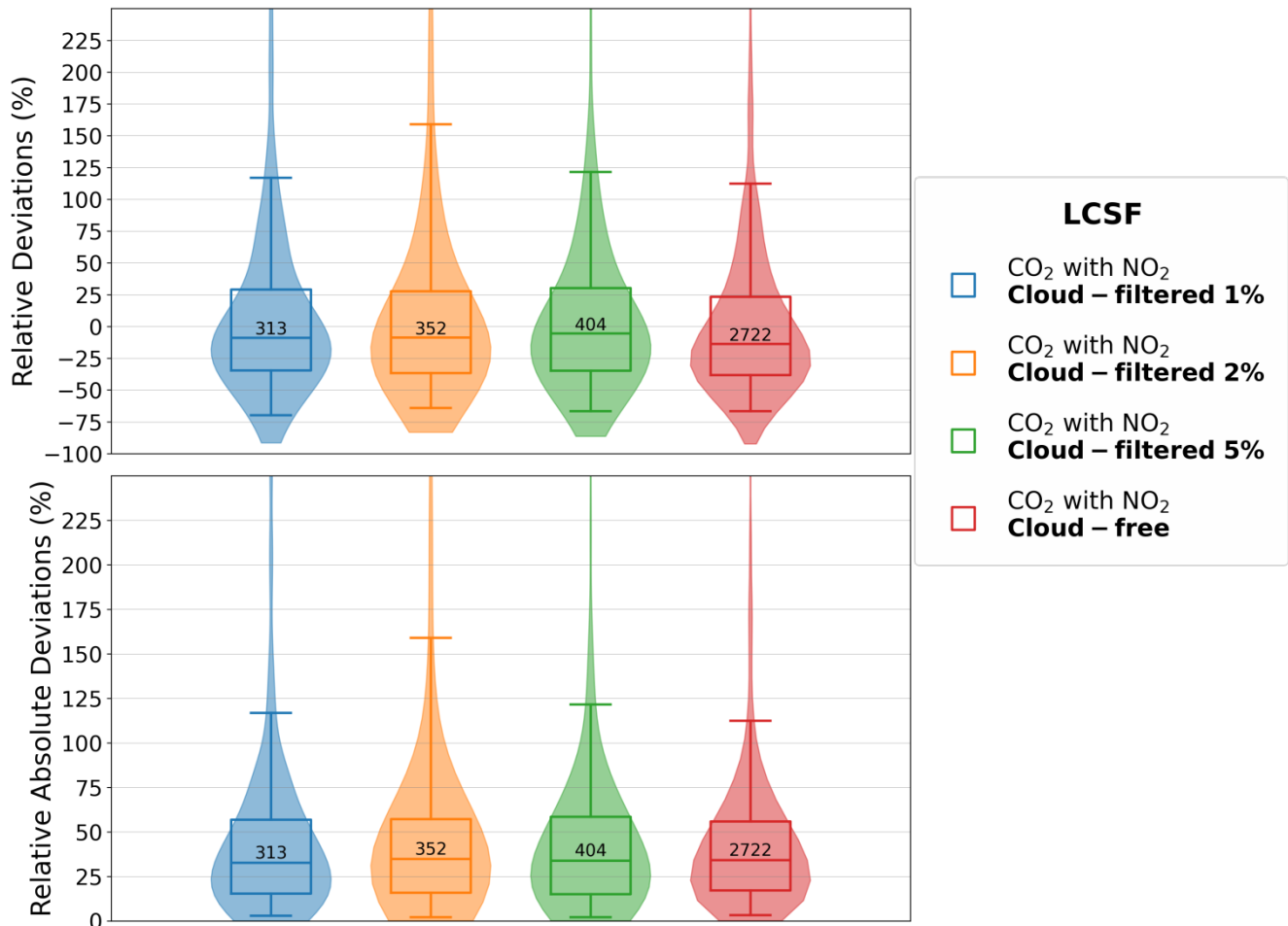
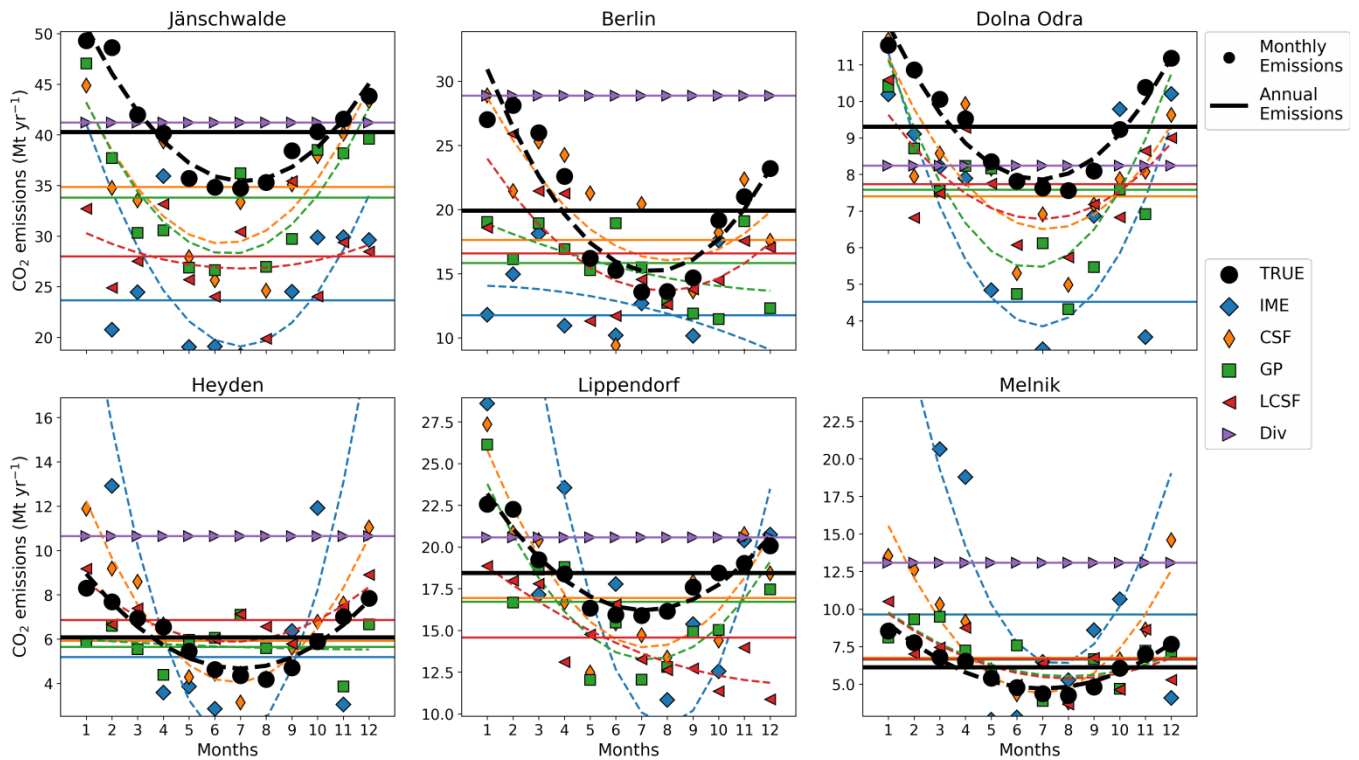


Figure A4: Performance of the LCSF method when estimating emissions from single images of CO<sub>2</sub> and NO<sub>2</sub> without considering clouds (in red) and for different cloudiness thresholds: 1% (in blue), 2% (in orange) and 5% (in green). Distributions of the relative deviations (top panel) and relative absolute deviations (bottom panel) are illustrated using violin plots. Boxes are the inter-quartiles of the distributions, the whiskers are the 5<sup>th</sup> and 95<sup>th</sup> percentiles, and the lines within boxes are the medians. Numbers in the inter-quartile boxes are the number of estimates for each benchmarking scenario.

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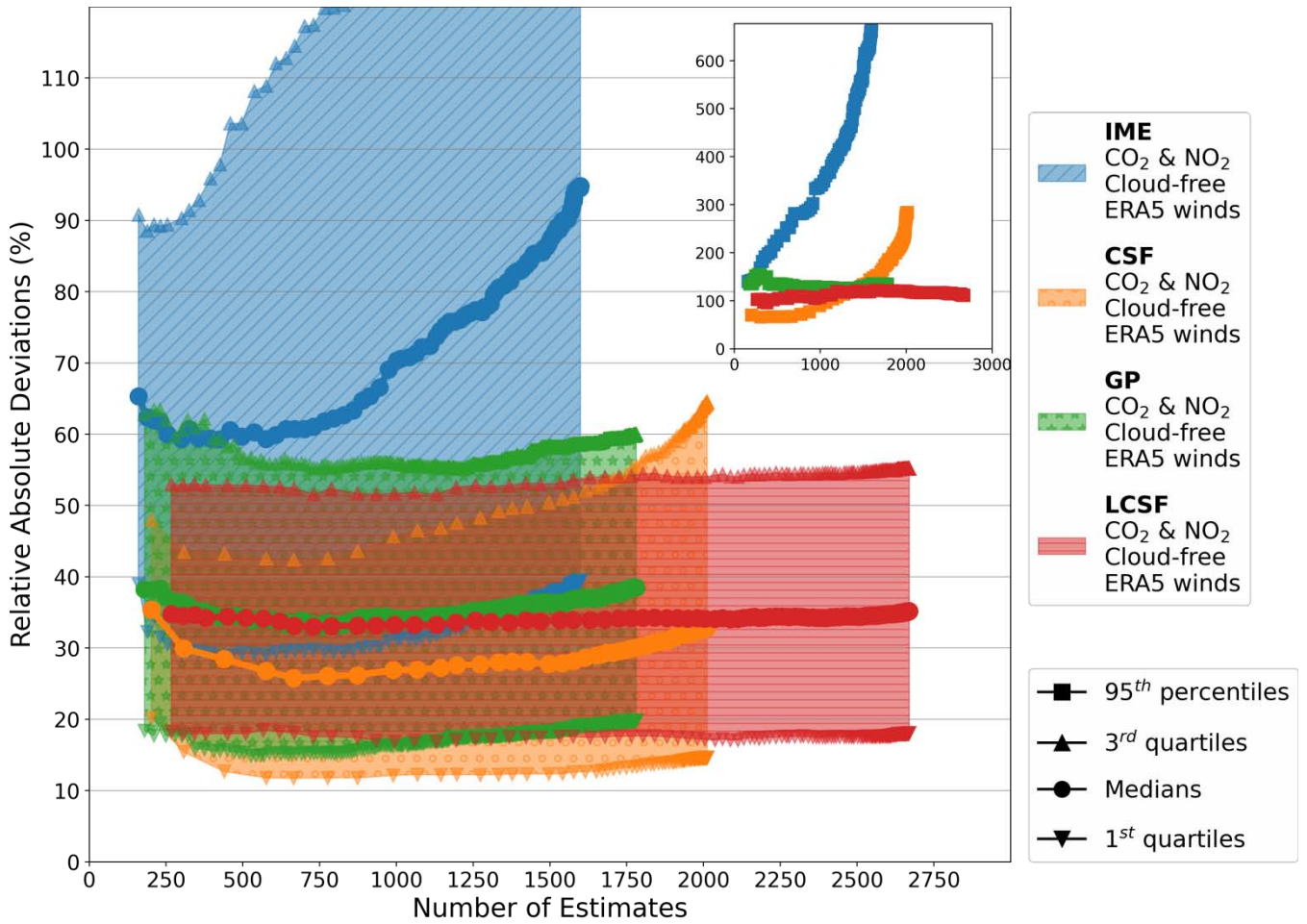
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**Figure A5: Annual and monthly estimates of the true and estimated emissions for different sources and for different inversion methods. Each panel is associated with a given source. Plain lines and markers represent annual averages and monthly averages respectively. Dashed lines represent the fits by a 2<sup>nd</sup> order polynomial of the monthly estimates. Colours are associated with different inversion methods (true emissions are in black). Annual and monthly estimates for the IME and CSF methods are weighted means of image estimates. Annual and monthly estimates for the GP and LCSF are means of image estimates while for the divergence method, we use the annual estimate also for monthly estimates. All inversion methods use CO<sub>2</sub> and NO<sub>2</sub> cloud-free data (CO<sub>2</sub> data only for the Divs methods) with ERA5 winds.**



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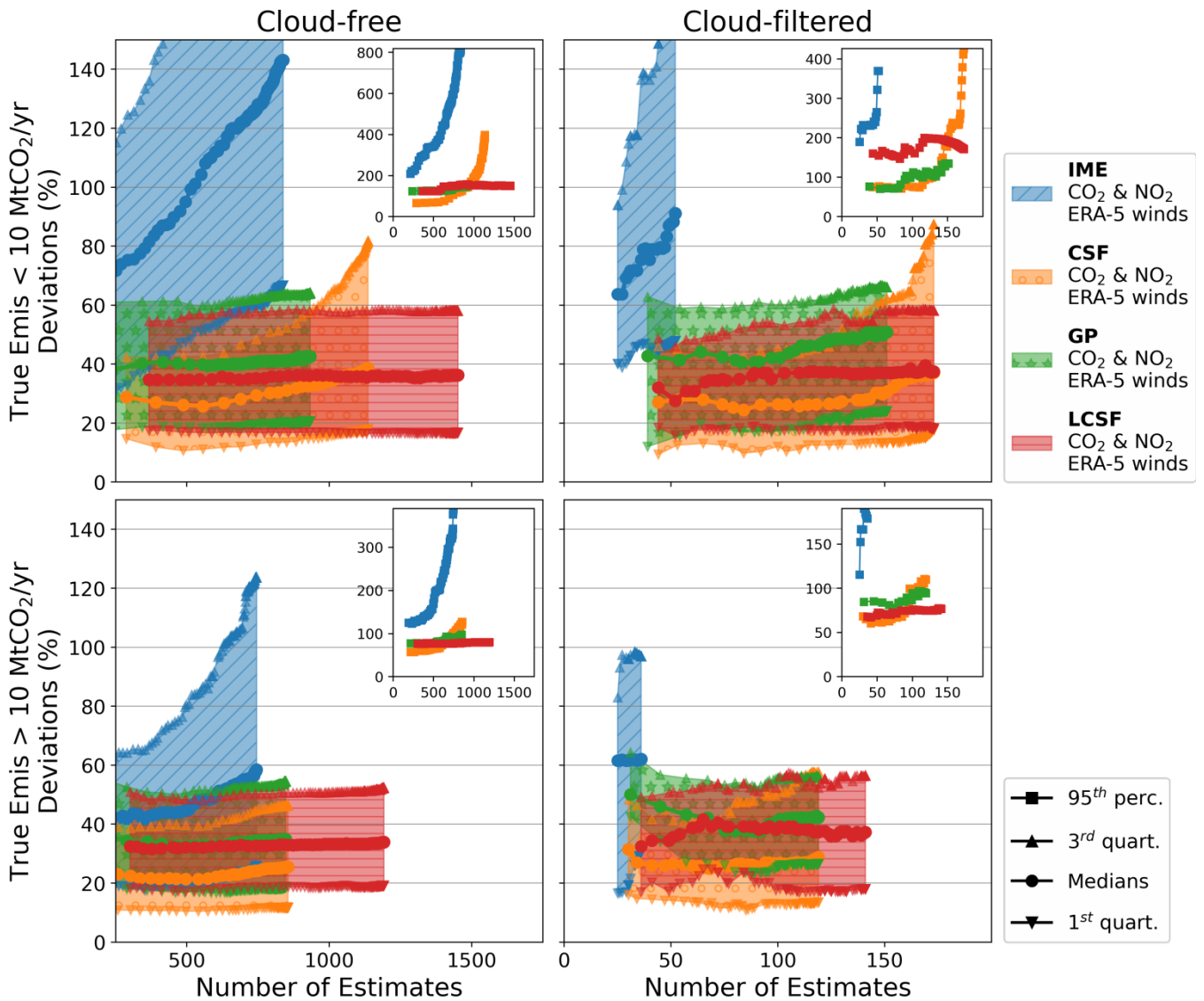
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**Figure A6. Accuracy of inversions vs number of instant estimates. The inversion methods shown here use CO<sub>2</sub> and NO<sub>2</sub> cloud-free data and ERA5 winds. The filled areas represent the inter-quartiles of the distributions of the relative absolute deviations depending on the number of estimates. The 95<sup>th</sup> percentiles of the distributions are represented in the inset. Points belonging to a same curve are associated to different QIs and from left to right along curves, points are associated with a decreasing QI; the points at the left and right ends of the curves are associated with the maximal and minimal QIs respectively.**



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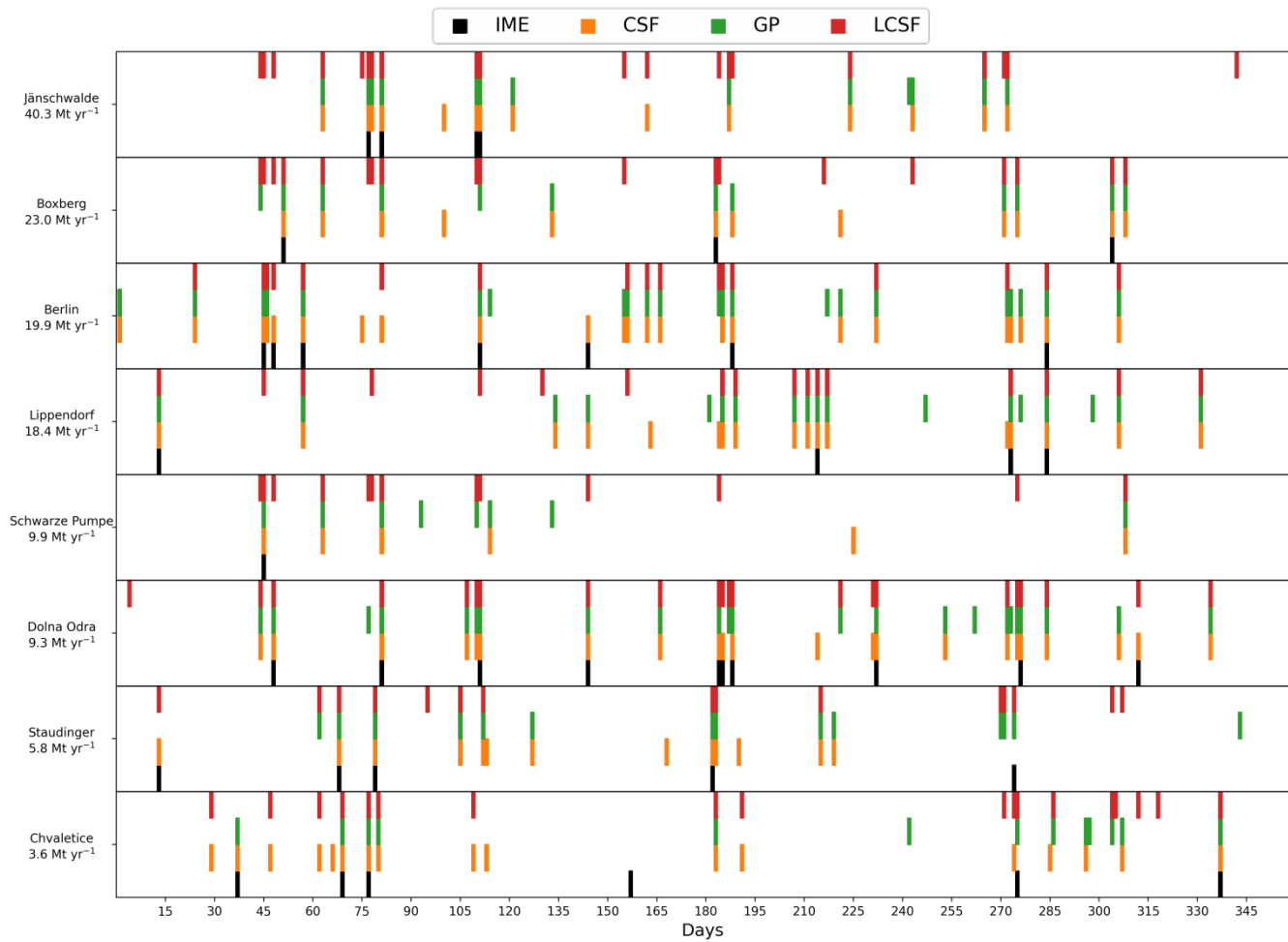
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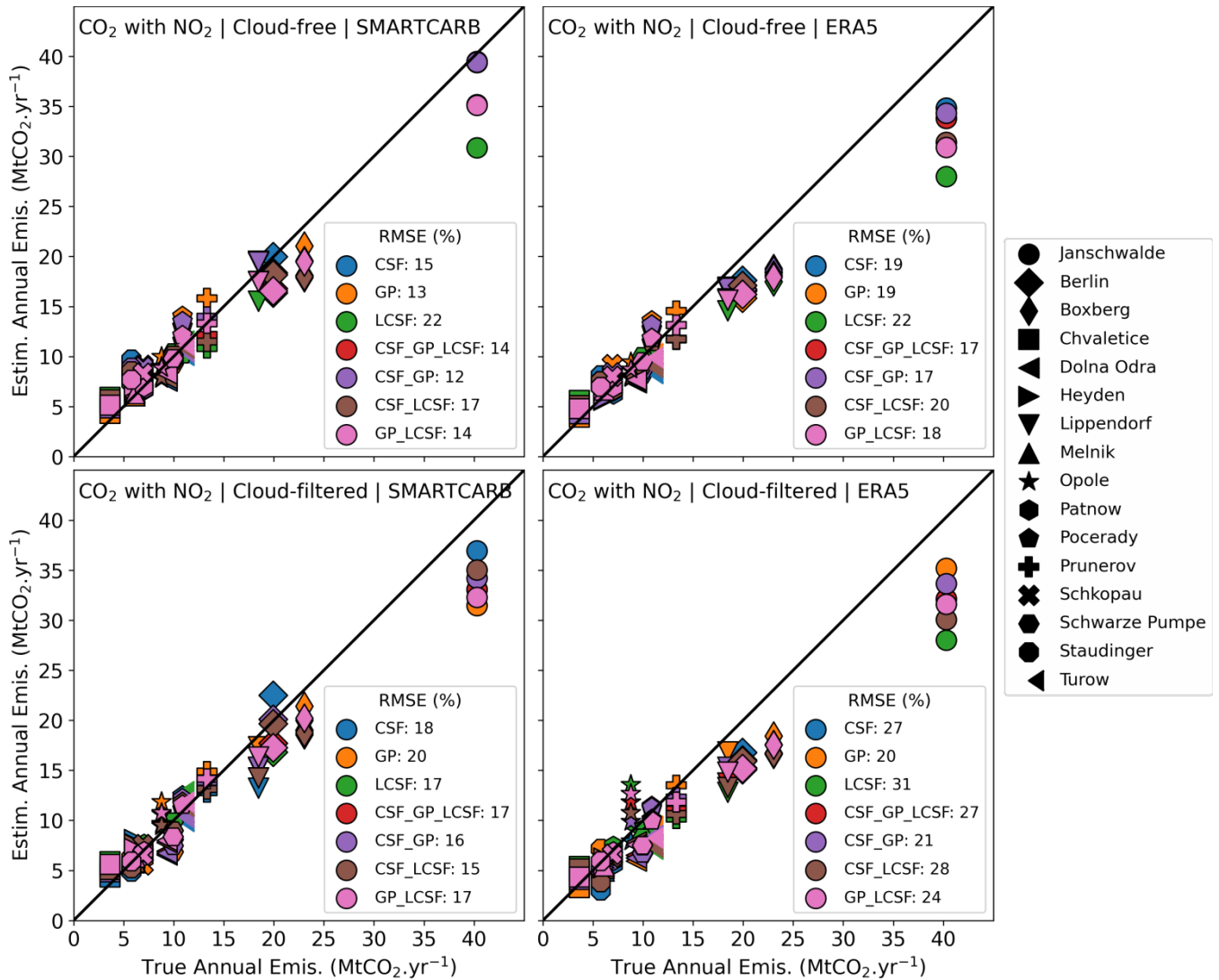
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**Figure A7: Accuracy of inversions vs number of instant estimates. The inversion methods shown here use CO<sub>2</sub> and NO<sub>2</sub> data, ERA5 winds and for cloud-free (1<sup>st</sup> column) and cloud-filtered data (2<sup>nd</sup> column). Results are shown for the cases where true CO<sub>2</sub> emissions of sources are below (1<sup>st</sup> row) and above (2<sup>nd</sup> row) 10 Mt yr<sup>-1</sup>. The filled areas represent the inter-quartiles of the distributions of the relative absolute deviations depending on the number of estimates. The 95<sup>th</sup> percentiles of the distributions are represented in the insets. Each point belonging to a same curve is associated with a different QI and from left to right along a same curve; points are associated with a decreasing QI.**



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**Figure A8:** Days of 2015 (x-axis) for which the IME, CSF, GP and LCSF methods produce estimates for the CO<sub>2</sub> emissions of eight sources (y-axis). For a given day, the availability of an estimate from a given inversion method is illustrated by a color bar (for color explanation, see legend of the figure). Inversions use CO<sub>2</sub> and NO<sub>2</sub> cloud-filtered data and ERA5 winds.



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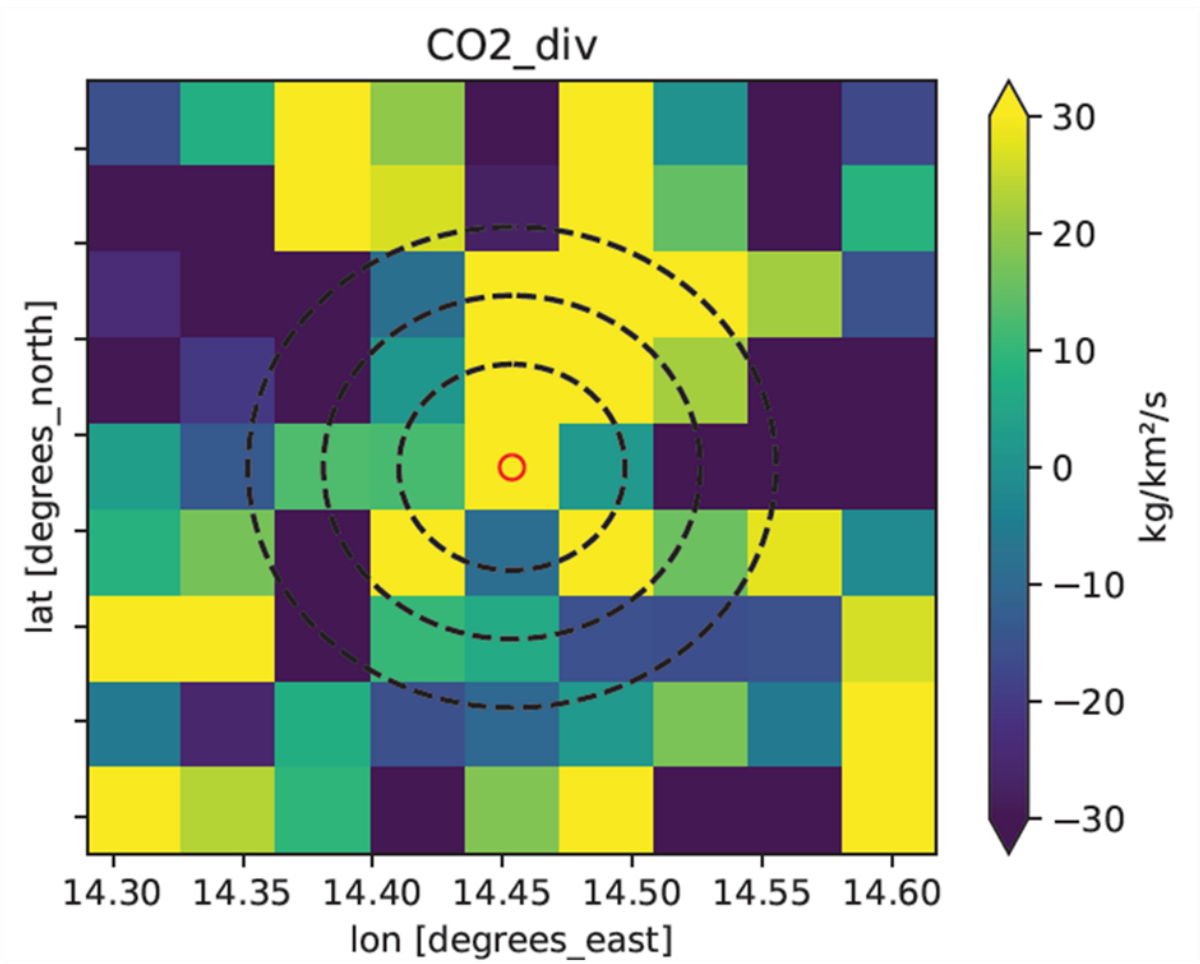
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**Figure A9: Estimated vs true annual emissions for 4 inversion scenarios (titles of the panels). Results are displayed for the CSF, GP, LCSF and ensemble methods that gather 2 or 3 of these individual methods. For the CSF method, annual estimates are weighted means of the instant estimates while they are arithmetic means for the GP and LCSF methods. Each marker represents a given emission source and each color a given inversion method. The divergence inversion method uses CO<sub>2</sub> data only for all the inversion scenarios. The plain line represents the 1:1 line. The bottom-right legends display for each inversion method the relative RMSE which is the RMSE between estimated and true annual emissions divided by the median of true annual emissions of all sources (~9.6 MtCO<sub>2</sub>.yr<sup>-1</sup>).**

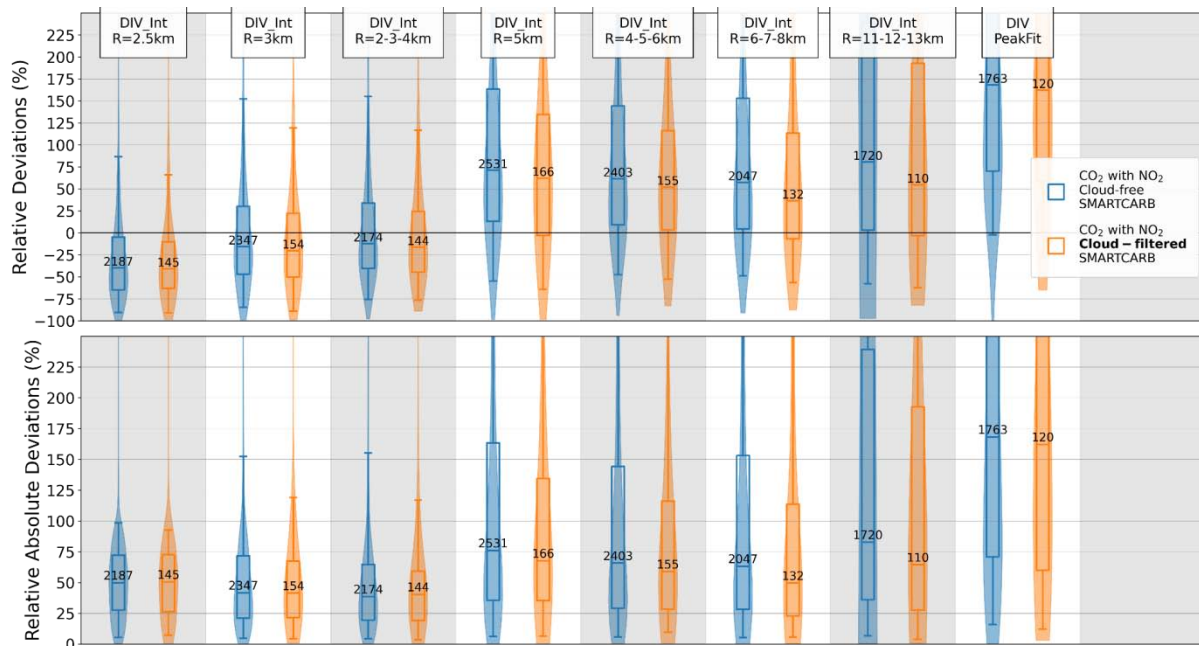
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Figure A10: Divergence map estimated around the Jämschwalde power station on January 2015 the 12th. Dotted circles show different radii (3 km, 5 km and 7 km) which define integration disks that could be used by the integral divergence method.



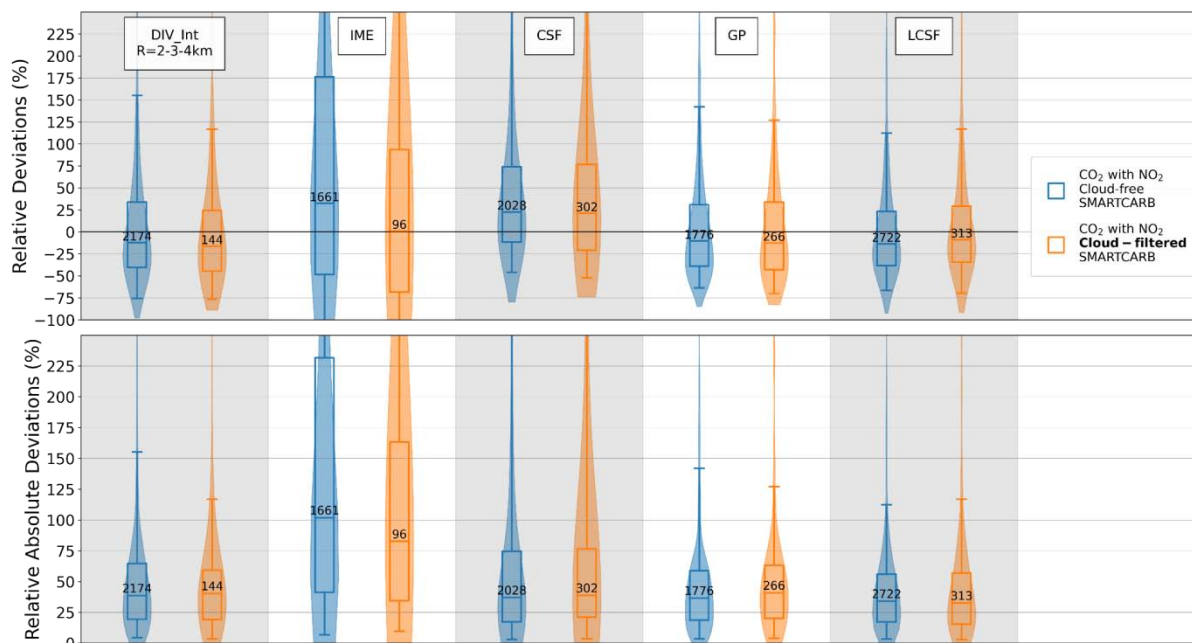
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1355 **Figure A11: Performances of the different versions of the divergence inversion method when estimating emissions from one year**  
 1356 **of single images for different benchmarking scenarios: cloud-free CO<sub>2</sub> and NO<sub>2</sub> data with SMARTCARB winds (in blue) and**  
 1357 **cloud-filtered CO<sub>2</sub> and NO<sub>2</sub> data with SMARTCARB winds (in orange). Distributions of the relative deviations (top panel) and**  
 1358 **relative absolute deviations (bottom panel) are illustrated using violin plots. Boxes are the inter-quartiles of the distributions, the**  
 1359 **whiskers are the 5<sup>th</sup> and 95<sup>th</sup> percentiles, and the lines within boxes are the medians. Numbers in the inter-quartile boxes are the**  
 1360 **number of estimates for each benchmarking scenario and inversion method. Methods DIV\_int\_R=xkm and DIV\_PeakFit are the**  
 1361 **integral (for an integration radius of x km) and peak-fitting versions of the divergence approach respectively. For a given overpass**  
 1362 **and source, the emission estimate of the method DIV\_int\_R=x-y-zkm is the average of the estimates when integrating over circles**  
 1363 **of x, y and z km radius around the source.**

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1365 **Figure A12: Performances of the inversion methods when estimating emissions from one year of single images for different**  
 1366 **benchmarking scenarios: cloud-free CO<sub>2</sub> and NO<sub>2</sub> data with SMARTCARB winds (in blue) and cloud-filtered CO<sub>2</sub> and NO<sub>2</sub> data**  
 1367 **with SMARTCARB winds (in orange). Distributions of the relative deviations (top panel) and relative absolute deviations (bottom**  
 1368 **panel) are illustrated using violin plots. Boxes are the inter-quartiles of the distributions, the whiskers are the 5<sup>th</sup> and 95<sup>th</sup>**  
 1369 **percentiles, and the lines within boxes are the medians. Numbers in the inter-quartile boxes are the number of estimates for each**  
 1370 **benchmarking scenario and inversion method. Methods DIV\_int\_R=2-3-4km and DIV\_PeakFit are the integral and peak-fitting**  
 1371 **versions of the divergence approach respectively. For a given overpass and source, the emission estimate of the method**  
 1372 **DIV\_int\_R=2-3-4km is the average of the estimates when integrating over circles of 2,3 and 4 km radius around the source.**

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Method	Time frame	Computational cost (1)
Integrated Mass Enhancement (IME)	Single-Image estimates	Medium: ~20 min
Cross-Sectional Flux (CSF)	Single-Image estimates	Medium: ~25 min

Gaussian Plume (GP)	Single-Image estimates	High: ~110 min
Light Cross-Sectional Flux (LCSF)	Single-Image estimates	Low: ~10 min
Divergence (Div)	Averaged estimates from ensemble of images	Medium: ~23 min

1375 **Table 1: Summary of characteristics of the benchmarked methods. (1) Computation time was estimated by inverting one month of**  
1376 **CO<sub>2</sub> and NO<sub>2</sub> cloud-free SMARTCARB data on the same server using the ddeq package (Kuhlmann et al., 2023)**

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Benchmark Scenario	Wind dataset	Cloud fraction thresholds	Joint use of NO <sub>2</sub> and CO <sub>2</sub>
Scenario 1	SMARTCARB	100% (no clouds)	Yes
Scenario 2	SMARTCARB	1% for CO <sub>2</sub> , 30% for NO <sub>2</sub>	No
Scenario 3	SMARTCARB	100% (no clouds)	No
Scenario 4	SMARTCARB	1% for CO <sub>2</sub> , 30% for NO <sub>2</sub>	Yes
Scenario 5	ERA5	100% (no clouds)	Yes
Scenario 6	ERA5	1% for CO <sub>2</sub> , 30% for NO <sub>2</sub>	No
Scenario 7	ERA5	100% (no clouds)	No
Scenario 8	ERA5	1% for CO <sub>2</sub> , 30% for NO <sub>2</sub>	Yes

1378 **Table 2: List of the different benchmarking scenarios: from the most optimistic (scenario 1) which considers inversions with cloud-**  
1379 **free data and SMARTCARB winds to the most realistic (Scenario 8) with cloud-filtered data and with ERA5 winds. Note that a**  
1380 **cloud fraction threshold of x% corresponds to the rejection of data pixels if their cloud cover exceeds x%, so that a cloud fraction**  
1381 **of 100% yields full images without a loss of data pixels.**

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Inversion method	Cloud-free data	Cloud-filtered data
IME	1661	96
CSF	2028	302
GP	1776	266
LCSF	2722	313

1385 **Table 3. Number of estimates for each inversion method when data with or without clouds are used. Inversions are**  
1386 **performed with CO<sub>2</sub> and NO<sub>2</sub> data and, with SMARTCARB winds.**

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