General comments:

The paper uses a symmetric rain rate to define the radar reflectivity error in the assimilation algorithm based on the symmetric rain rate referring to the symmetric error model in satellite all-sky assimilation. The paper is well-structured but still, there are many ambiguous sentences in the paper which need to be rewritten/clarified.

Authors appreciate the constructive comments from referee #3. Many operation centers, such as ECMWF, NCEP, ECCC and CMA, have reported the positive impacts of using the symmetric error model in all-sky satellite radiance assimilation. Although those symmetric error models have been built on the basis of different NWP models and assimilation systems, even different symmetric predictors, authors noticed that the procedure for building the symmetric error model is the same. Moreover, the reflectivity assimilation suffers similar issues in all-sky satellite radiance assimilation. Thus, authors demonstrated the symmetric error model can attack the non-Gaussian problem in radar reflectivity assimilation.

After carefully considering the comments left by all referees, including previous reviewers from *Earth and Space Science*, authors think this study better entitles "*Improving the Gaussianity by Using the Symmetric Rain Rate Toward Reflectivity Assimilation*" and hope this study could engage the readers who can build the symmetric error model based on their own NWP models and assimilation systems.

Authors reply all comments from Referee #3 and explained our ideas in the following blue words.

major revisions:

1• The very important point that is missing in the paper is that the reflectivity error in an assimilation algorithm is a representative error based on each assimilation system/algorithm meaning that if anything changes in the assimilation algorithm either the NWP model or the number/type of observations, the reflectivity error need to be recalculated /modified. But in this paper, it seems there is not any assimilation algorithm/system. The equivalent reflectivity is based on the 6-hour model forecast which does not represent the equivalent reflectivity after assimilation. The very large standard deviation (up to 35 dbz) clearly shows this inconsistency.

Response:

Authors agree the comment that the representative error in reflectivity assimilation is highly related to a certain NWP model and assimilation algorithm. The symmetric error model reported by this study may be inappropriate if the NWP system is changed. It is the reason that this study mainly focuses on how to build the symmetric error model based on observation and simulation data sets, which is rarely discussed before and may fall better in the scope of *Atmospheric Measurement Techniques*. Authors also expect that this study engages radar assimilation experts who want to build their symmetric error models by following the procedure reported in this study.

For the large inconsistency between observations and simulations, authors would like to brief the usage of the symmetric error model. Similar to the all-sky satellite radiance assimilation, the reflectivity error (σ, unit: dBZ) is inflated from a lower boundary to an upper boundary:

$$\sigma = \begin{cases} \sigma_l & RR_{avg} < RR_{avg1} \\ \sigma_l + \alpha\beta(RR_{avg} - RR_{avg1}) & RR_{avg1} \le RR_{avg} < RR_{avg2} \\ \sigma_u & RR_{avg2} \le RR_{avg} \end{cases} \quad \text{(R1)}$$

where RR_{avg} means the symmetric rain rate, σ_l and σ_u are the lower and upper boundaries of reflectivity error respectively, β is the slope of fitting functions and α is a tuning parameter. By tuning the parameter α, the representative error can either be assigned completely by the symmetric error model ($\alpha = 1$) or ignored ($\alpha = 0$). Tuning the parameter α, as designed by Geer and Bauer (2011), can improve applicability of the symmetric error model. Besides, tens of dBZ of reflectivity departures between observations and 6 hour forecast is common for convective NWP model.

Reference:
Geer, A. J., and Bauer, P.: Observation errors in all-sky data assimilation. *Q J R Meteorol Soc*, 137, 2024-2037, https://doi.org/10.1002/qj.830, 2011.

2• Besides, defining a better reflectivity error is supposed to improve the assimilation results. However, the paper did not show any plots related to applying the newly defined reflectivity error in an assimilation algorithm and the comparison with the constant error (which was claimed in the paper is not suitable for radar assimilation).

Response:

According to the PDF distributions (Fig. 10 and 11) and JSD (Table 3) in this manuscript, the symmetric error model can improve the raw non-Gaussian distribution of reflectivity error. At least in theory, a more Gaussian distribution is more consistent with most current data assimilation algorithms. Moreover, applying the symmetric error model on reflectivity assimilation could be very complicated, because some empirical parameters in Eq. (R1) should be discussed by data assimilation experiments. Authors will add the Eq. (R1) in our revision in order to clarify how to apply the symmetric error model on reflectivity. Considering the length and innovation, authors will not add any data assimilation experiment and still focus on how to build the symmetric error model and its impacts on PDF.

3• Line 92: What is the purpose of defining the radar composite based on the vertical maximum reflectivity? Do you use this composite in your assimilation algorithm? Why not the radar reflectivity composites of a specific level (which will be used later in the assimilation algorithm)?

Response:

Compared with the radar reflectivity composites of a specific level, the vertical maximum reflectivity not only represents the strength of convections, but provides more samples. Authors do not assimilate the vertical maximum reflectivity.

Although the definitions of composite reflectivity and rain rate have distinct differences, they are good indicators of convective storms. The strength and distribution of composite reflectivity and rain rate are associated with the variation of convective systems. The following Figure R1 shows the absolute correlations between the composite reflectivity and various rain rate data sets in the six months. For the derived rain rate data sets, most cases show the absolute correlations are about 0.75, despite some cases present low correlations. Thus, we argue that the composite reflectivity and derived rain rate are comparable for a precipitating weather system and the derived rain rates can be used to describe the heteroscedasticity of composite reflectivity in statistics, similar to the cloud liquid water or liquid water path for satellite radiances.

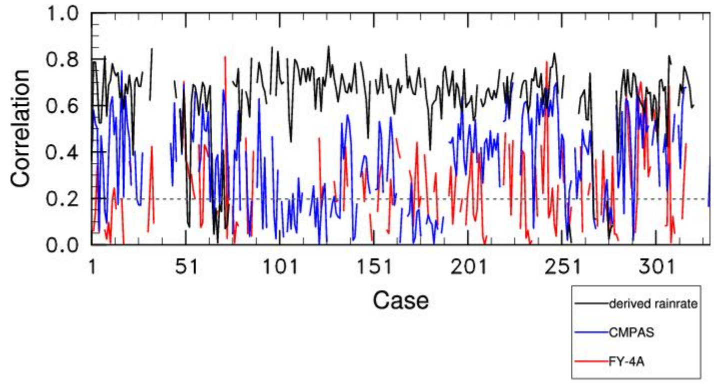

Figure R1. the absolute correlations between the composite reflectivity and various rain rate data sets in the six months. The black, blue and red lines represent the rain rate derived from reflectivity at 3 km altitude, the CMPAS data sets and the FY-4A data sets. The dash line shows the 95% confidence.

The absolute correlations of CMPAS and FY-4A decrease obviously because the independent errors from the third-party data sets, including the sampling and representative errors, increase rapidly. However, all averaged absolute correlations, which are 0.62, 0.35 and 0.24 for derived rain rate, CMPAS and FY-4A data sets respectively, can pass the 95% confidence. We argue that the differences among the three rain rate data sets allow us to investigate how the accuracy of predictor affects the symmetric error model.

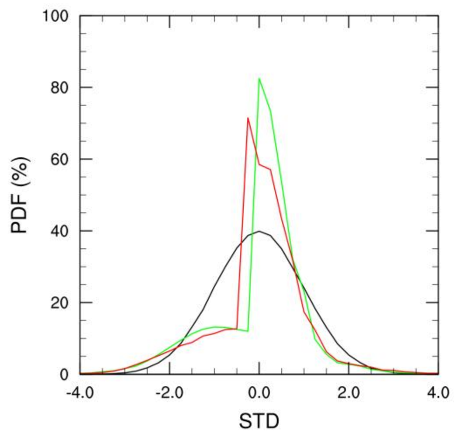

Figure R2. Probability distribution functions (PDFs) of OmBs (observations minus backgrounds) at 1 km altitude normalized by the standard deviation of the whole samples (green line) and by the symmetric error model (red line). the black line represents the normal Gaussian PDF.

Authors also employed the radar reflectivity composites at 1 km altitude to investigate the PDF as shown in Fig. R2. Compared with Fig. R4, the PDFs of 1 km reflectivity are similar to those of the vertical maximum reflectivity composite. It illustrates that the symmetric error models of the vertical maximum reflectivity and the radar reflectivity composites of a specific level are similar. Authors also found the symmetric error model built by the radar reflectivity composites at 1 km altitude is similar to that built by the vertical maximum reflectivity.

Thus, the piecewise function (Eq. R1) fitted by the vertical maximum reflectivity and the derived rain rate can be used to inflate the observation error at some specific levels, which are lower than the melting level.

4• It was mentioned that to match with the rain rate resolution of 4 or 5 km, the reflectivities with the resolution of 1 km were interpolated! Actually, this procedure is extrapolation. However, the normal trend in the radar assimilation is defining/using the super observation which defines the reflectivity over a larger grid box (compared to the original grid box) to match with other observations in the assimilation algorithm which have lower resolution.

Response:

It should be interpolation because the domain with 1 km resolution is larger than the domain with 5 km resolution. The values in coarse grids (red dash lines) were computed by the adjacent four points in fine grids (black solid lines), as shown in Figure R2. This data thinning strategy is similar to the super observation.

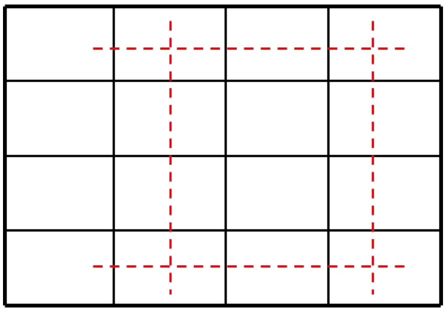

Figure R3. the schematic of thinning 1 km reflectivity.

This comment reminds authors that the zero reflectivity is usually not assimilated. For rainy echo, reflectivities larger than 5 dBZ are assimilated. Thus, the reflectivites smaller than 5 dBZ should be excluded when computing the PDF and building the symmetric error model. The Fig. R4 shows the PDF of OmBs of the vertical maximum composite reflectivity. Compared to the PDF of ‘any-reflectivity’ in Fig. 4, the PDF (green line) becomes an unimodal distribution. The negative peak disappeared because observations smaller than 5 dBZ are removed. Authors think this PDF distribution is closer to the reality in current reflectivity. The high peak is reduced if OmBs normalized by the symmetric model (red line). All reflectivity data sets are updated and authors will re-investigate the symmetric error model in revision.

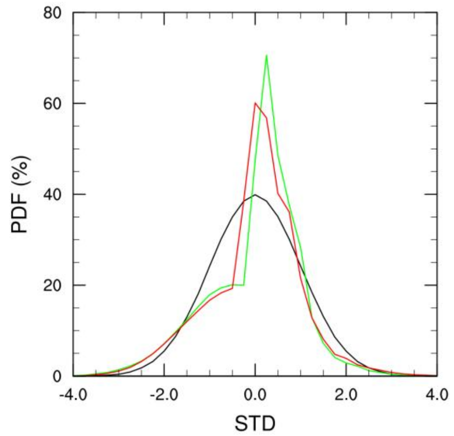

Figure R4. Probability distribution functions (PDFs) of OmBs (observations minus backgrounds) of the vertical maximum composite reflectivity normalized by the standard deviation of the whole samples (green line) and by the symmetric error model (red line). the black line represents the normal Gaussian PDF.

5 • Fig 2: It clearly can be seen that the simulated reflectivity and the observed reflectivity are far from each other. This means the model has a very poor ability to capture the convective events. As I mentioned, this could not be the representative reflectivity of an assimilation algorithm. It can be easily seen that defining the equivalent reflectivity based on this kind of plot can cause a high value for the reflectivity standard deviation which is not realistic.

Response:

Nowadays, many researches show near perfect simulations, but distinct disagreements between observations and simulations, such as the poor shape, strength and location, are common cases in daily operation. Because the NWP model and its initial data have limited ability to describe the distributions and variations of hydrometeors. The OmBs (observations minus backgrounds) often increase to tens of dBZ in 6 hours simulation, let alone the error of reflectivity operator.

Admittedly, using poor simulations exaggerates the representative error. The tuning parameter α in Eq. (R1) may remedy it by reducing the slope of fitting function β.

6 • Fig3: The two plots look identical. It couldn't/shouldn't be like this. Please check the plots.

Response:

The two plots are different on the abscissa and ordinate, which illustrate the impact of misses and false simulations on PDF in Fig. 4. However, according to the fourth comment, authors will re-do the construction of symmetric error model using the observations larger than 5 dBZ in the revision. Because it is more consistent with the reality of reflectivity assimilation.

7• What is the purpose of excluding the false and missed events? and defining the 'both reflectivity'? At the end, they all need to be included in defining the standard deviation.

Response:

Excluding the false and missed events is used to illustrate what give rise to the non-Gaussian error distribution. To avoid misunderstandings, authors will remove all plots about the 'both reflectivity' from Section 3. The construction of symmetric error model will be only based on the 'any reflectivity' in revision.

8• As it was written the CMPAS rain rate is the most complete one (or maybe the reference ones) what is the purpose of using another product (FY-4A)?

Response:

The essential point of earlier symmetric error models in all-sky microwave radiance assimilation is that the hydrometeor predictor is derived from the radiances themselves, either the observations or the equivalent radiance simulations. Thus, the derived rain rate is the reference in this study.

Authors attempted to use third-party rain rate data sets to replace the derived rain rate data sets, which is a major different from previous works. As shown in Fig. R1, the CMPAS rain rate is more accurate, but its correlation is lower than the derived rain rate. Because the third-party rain rate has its own representation and mislocation errors. However, authors attempted to argue that the third-party rain rate could be still useful in construction of symmetric error model, if the independent error of the third-party is small. the third-party rain rate cannot be used if the independent error is large, which is the case of FY-4A.

Authors can remove the plots and discussions related to FY-4A data sets since it is far away the aim of this study.

9• In Fig 6 (or later in Fig 8), the departure or the standard deviation needs to be defined clearly. It is not clear that the departure means 'obs-model' or 'model-obs'.

Response:

All departures are OmB (observations minus backgrounds). Authors will clearly define it in revision.

10• Fig 8 (Line 258): It was mentioned that the standard deviation increases after 9 mm h^{-1} because of the error in the WRF model or the initial data. Actually, a big part of the error could be due to the deficiency in the forward model (the process of converting the model state to the radar reflectivity).

Response:

Authors agree this comment. The error of observation operator could become large when ice-phased hydrometeors exist. The observation operator is single moment, which is based on rain, snow, and graupel mixing ratios was designed by Stoelinga (2005):

$$Z = 10\log_{10}(Z_{er} + Z_{es} + Z_{eg}) \quad \text{(RR0)}$$

Following some assumptions, the reflectivity contributed by rain droplets is given by:

$$Z_{er} = \Gamma(7) N_{r0} \lambda_r^{-7} \quad \text{(RR1)}$$

$$\lambda_r = (\frac{\pi N_{r0} \rho_l}{\rho_a q_{ra}})^{1/4} \quad \text{(RR2)}$$

where N_{r0} is 8×10^6, ρ_l and ρ_a are the liquid water density and dry air density respectively and q_{ra} is the rainwater mixing ratio in background.

Assumed snow particles are spheres, the reflectivity contributed by snow is given by:

$$Z_{es} = \alpha\Gamma(7) N_{s0} (\frac{\rho_s}{\rho_l})^2 \lambda_s^{-7} \quad \text{(RR3)}$$

$$\lambda_s = (\frac{\pi N_{s0} \rho_s}{\rho_a q_{sn}})^{1/4} \quad \text{(RR4)}$$

where α is 0.224, N_{s0} is 2×10^7, ρ_s is the density of snow 100 kg m^{-3} and q_{sn} is the snow water mixing ratio in background.

Similarly, the contribution of graupel particles can be obtained:

$$Z_{eg} = \alpha\Gamma(7) N_{s0} (\frac{\rho_g}{\rho_l})^2 \lambda_g^{-7} \quad \text{(RR5)}$$

$$\lambda_g = (\frac{\pi N_{g0} \rho_g}{\rho_a q_{gn}})^{1/4} \quad \text{(RR6)}$$

where α is also 0.224, N_{g0} is 2×10^7, ρ_g is the density of graupel 400 kg m^{-3} and q_{gn} is the graupel water mixing ratio in background. This algorithm can be used as the forward operator in reflectivity assimilation. Similar forward operator of reflectivity based on double-moment Thompson microphysics was employed by Liu et al. (2022). In future, the impacts of observation operator on the symmetric error model should be discussed.

Authors will add some sentences in revision to illustrate the deficiency in the forward model

Reference:

Liu, C., H. Li, M. Xue, Y. Jung, J. Park, L. Chen, R. Kong, and C. Tong, 2022: Use of a Reflectivity Operator Based on Double-Moment Thompson Microphysics for Direct Assimilation of Radar Reflectivity in GSI-Based Hybrid En3DVar. Mon. Wea. Rev., 150, 907–926, https://doi.org/10.1175/MWR-D-21-0040.1.

11• Fig8: When there is (kind of) reference data set of rain rate, why the standard deviation should be plotted based on the derived rain?

Response:

Same to the fifth comment, the derived rain rate is the reference data set in the symmetric error model. The CMPAS and FY-4A data sets were used to discuss the impacts of the third-party observations on the symmetric error model.

12• Line 279 or Line 283: What was here exactly normalized? Fig 8 and Fig 9 are the standard deviations based on the rain rate. Did you normalize any variables here?

Response:

In Fig. 8 and Fig. 9, each OmB bin, 0.5 mm h^{-1} interval, is normalized separately, i.e. OmBs of reflectivity are normalized by different standard deviations, instead of normalization by the standard deviation of the whole samples.

This is the artful normalization to attack the non-Gaussian PDF. Although the PDF of whole samples is not Gaussian, the whole samples can be separated by many sub-groups with Gaussian PDF. Thus, the observation errors in reflectivity can be estimated separately.

13• Fig 8: The black dashed line shows the log of sample numbers that reach less than 2 after 15 mm h^{-1} meaning that the number of samples is less than 100! If this is the case, means that the number of samples in these bins is not enough to do the standard deviation. As it was shown the number of samples in some bins can reach 10^6 so there is a big inconsistency in defining the standard deviation in different bins. There should be a limit based on the number of samples to do the standard deviation. I would suggest a limit of 10^3 or 10^4.

Response:

Authors could not agree this comment more. The linear regression is lower than 9 mm h^{-1}, where the sample number is larger than 10^3. Thus, the piecewise function is a constant when the sample number is smaller than 10^3. Authors will emphasize this point in revision.

minor correction:

• Line 76: "stage IV precipitation" → What is the defintion of stage IV precipitation?

Response: the stage IV precipitation is a precipitation data set produced by NCEP. We mentioned it here to support the high correlation between reflectivity and precipitation.

• Line 99: "on May 3th 2021" → on 3rd of May 2021

Response: authors will correct this mistake in revision.

• Line 218: "…, chosen to be larger than 100 samples": What does this mean? If it means that you calculate the reflectivity departure for the grid box which has more than 100 samples, why the colorbar starts from 1?

Response: this sentence attempted to describe that the results is credible when the samples larger than 100. Authors will revise this sentence in revision.

• Line 219: "excessive reflectivities" → overestimation in simulating the reflectivities

Response: authors will rewrite this sentence in revision.

- Line 224: "It could be argued" → It shows that

Response: authors will rewrite this sentence in revision.

- Line 235: "exists" → defines the relation between …

Response: authors will rewrite this sentence in revision.

- Line 257: What is the "geophysical boundary"?

Response: it means the boundary in the map, such as the boundary between rainy and non-rainy areas.

- Line 262: The constant reflectivity error is usually between 5 and 10 dbz

Response: the constant reflectivity error is empirical. Authors have reviewed several constants in different researches. Authors will rewrite 5 or10 dBZ in revision.

- Line 323: "… from the large mislocation errors in the main" → What is main?

Response: this sentence attempted to explain the large mislocation errors are the main reason of non-Gaussian distribution. Authors will rewrite this sentence in revision.