First and foremost, we would like to express our sincere gratitude to Luca Lelli, the anonymous reviewers, the editor, and the editorial support team for taking the time to review our manuscript and provide valuable feedback. The comments we received were extremely helpful in improving our manuscript, and we are very grateful for them. As outlined below, we have revised the manuscript based on the feedback. The reviewers' comments are copied below and shown in *italics*, while our responses and the corresponding text in the manuscript are shown in red and orange, respectively.

Response to the editorial support team

Regarding figures 3, 7: please ensure that the colour schemes used in your maps and charts allow readers with colour vision deficiencies to correctly interpret your findings. Please check your figures using the Coblis – Color Blindness Simulator (https://www.color-blindness.com/coblis-color-blindness-simulator/) and revise the colour schemes accordingly with the next file upload request.

Answer: In response to the comment, we updated the color scheme for Figures 3 and 4 (excluding Figure 3a) to the 'Scientific Color Maps' recommended on the AMT submission page (https://www.atmospheric-measurement-techniques.net/submission.html).We recognize that adjusting the color scheme of the RGB images in Figures 3a and 7 as well would also be preferable. However, since the values of the three channels are directly assigned to R, G, and B, we are unsure how to modify them to make them colorblind-friendly. Instead, we utilized the 'Coblis – Color Blindness Simulator' to confirm that the RGB images in Figures 3 and 7 can be correctly interpreted by readers with anomalous trichromacy.

Response to Anonymous Referee #3

A new algorithm was proposed to retrieve the CBH using the SGLI 763 nm channel in combination with several other SGLI channels in the visible, shortwave infrared, and thermal infrared regions. However, there are some critical aspects that require more detailed elaboration and clarification to enhance the clarity of your findings.

We would like to thank you very much for carefully reading our manuscript and providing us with valuable comments. We have revised our manuscript, by taking full account of the referee's suggestions. The original comments are copied below and shown in *italics*, while our responses and the corresponding text in the manuscript are shown in red and orange, respectively.

1. *The optimal estimation algorithm is undoubtedly the heart of your study. Please expand on the methodology section to provide a comprehensive and step-by-step description of the algorithm. This should include the mathematical formulations, assumptions made, and any pre-processing or post-processing steps involved. This will enable readers to fully understand your work.*

Answer: In accordance with the comment, the description of the optimal estimation method using the Levenberg–Marquardt (LM) approach has been revised and elaborated as follows in Section 2.2.1 of the revised manuscript:

[Section 2.2.1; Lines 194 - 209]

"Our algorithm employs the optimal estimation method framework (Rodgers, 2000), which seeks the optimal solution that minimizes the cost function J that is given by the following equation:

$$J = [\boldsymbol{y} - \boldsymbol{F}(\boldsymbol{x})]^T \boldsymbol{S}_e^{-1} [\boldsymbol{y} - \boldsymbol{F}(\boldsymbol{x})] + [\boldsymbol{x} - \boldsymbol{x}_a]^T \boldsymbol{S}_a^{-1} [\boldsymbol{x} - \boldsymbol{x}_a], \quad (1)$$

where $\boldsymbol{y}$ and $\boldsymbol{F}(\boldsymbol{x})$ represent the measurement vectors consisting of the measured and simulated TOA reflectances, respectively. $\boldsymbol{x}$ represents the state vector, $\boldsymbol{x}_a$ represents the a priori values for $\boldsymbol{x}$, and $\boldsymbol{S}_e$ and $\boldsymbol{S}_a$ represent the covariance matrices for $\mathbf{y}$ and $\boldsymbol{x}_a$, respectively.

The iterative solution of the inverse problem through the Levenberg–Marquardt approach, based on the Gauss-Newton method for minimizing Equation (1), is determined using the following formula (Rodgers, 2000):

$$x_{i+1} = x_i + \left[(1+\gamma)\, S_a^{-1} + K_i^T S_e^{-1} K_i\right]^{-1} \{K_i^T S_e^{-1}[y - F(x_i)] - S_a^{-1}[x_i - x_a]\}, \quad (2)$$

where x_i is the state vector at the i-th iteration, K_i is the Jacobian matrix of $F(x)$ evaluated at x_i, γ is a parameter chosen at each step of the iteration to reduce the cost function J."

2. *Clarify how you address the non-Gaussian distributions of the observations. Discuss the limitations and potential biases introduced by these assumptions.*

Answer: The optimal estimation method (Rodgers, 2000) in Formula (1) is formulated based on the assumption that the a priori distributions for the measurement vector and state vector (i.e., S_e and S_a) follow Gaussian distributions. Consequently, non-Gaussian distributions are not considered in this study.

We believe it is reasonable to assume a Gaussian distribution for measurement errors of TOA radiances (i.e., S_e) that has been accurately calibrated radiometrically and exhibits sufficiently small measurement errors. However, for the prior distribution of the state vector (i.e., S_a), non-Gaussian distributions may be more appropriate in certain circumstances. For instance, the log-normal distribution might better fit the histogram of cloud optical thickness. However, since this is the first application of our algorithm to GCOM-C/SGLI, we used a normal distribution for simplicity with means of typical orders of magnitude and fairly large standard deviations to avoid excessive reliance on the prior distribution. To clarify these points, the following text has been added.

[Section 2.2.1; Lines 221 - 225]

"Note that the values in Table 1 could be assigned more appropriate prior distributions (mean, standard deviation, and even covariance) by using cloud property products from other satellite observations. However, since this is the first application of our algorithm to GCOM-C/SGLI, we used a normal distribution for simplicity with means of typical orders of magnitude and fairly large standard deviations to avoid excessive reliance on the prior distribution."

3. *Provide details on how you estimate and incorporate the covariance matrix of the observations, particularly addressing the correlations between different channels. Discuss any challenges in estimating these correlations and the strategies employed*

to mitigate their impacts on the estimation accuracy.

Answer: For the covariance matrix of the measurement vector $\boldsymbol{S_e}$, the diagonal elements (i.e., variances), which represent uncertainties of measurements for each channel, were given based on the post-launch calibration information of the GCOM-C mission. Meanwhile, the non-diagonal elements (i.e., covariances), which represent correlation of measurement errors between channels, were all set to zero. To clarify these points, we have revised the text as follows:

[Section 2.2.1; Lines 216 - 218]

"The diagonal elements of $\boldsymbol{S_e}$, consisting of the uncertainties in the TOA measurements, were obtained from the post-launch calibration information of the GCOM-C mission, while the non-diagonal elements of $\boldsymbol{S_e}$ were set to zero."

In general, issues that cause significant bias across multiple channels (e.g. electrical leakage and/or stray light) are identified and addressed during the sensor development stage. For GCOM-C/SGLI, no such critical problems have been reported since its launch. Additionally, radiance calibration after launch is typically performed independently for each channel, providing the uncertainty (i.e., variances) of the radiance for each channel, but it does not usually provide the correlation (i.e., covariance) of the measurement errors between channels. It may be possible to estimate the covariance of measurement errors through vicarious calibration using in-situ measurements or mutual comparisons with other satellite sensors; however, the reliability of such estimates is low. In conclusion, we consider it challenging to assign a value to the correlation of measurement errors for $\boldsymbol{S_e}$ and deem it most appropriate to assume it as 0.

4. *Explain how you account for angular biases or other systematic errors in the observations, particularly as they relate to the state variables.*

Answer: In the optimal estimation method, systematic biases such as angular biases are typically addressed by adding them collectively to S_e, rather than considering them individually. However, if all possible systematic biases were to be directly added together, S_e will become too large relative to S_a,, causing the solution to be overly constrained to x_a. Therefore, as described in the main text, we determined S_e based on the post-launch calibration information for GCOM-C, while assigning relatively loose prior distributions to S_a as shown in Table 1, and designed the algorithm to be tightly constrained by the SGLI measurements rather than by the a priori information.

5. *Elaborate on the methodology used to determine the background error covariance for the state variables. Specifically, discuss how you handle correlations between different state variables and how you arrived at the values presented in Table 1. Consider discussing the sensitivity of your results to these assumptions and any validation performed to support the chosen values.*

Answer: Please allow us to partially reiterate our response to Comment 2 above in addressing this comment. The values in Table 1 were provided roughly based on cloud property products from other satellite observations, without overly constraining the solution space. For the optimal estimation method to be most effective, a prior distribution close to the true value should be used. However, since this is the first application of our algorithm to GCOM-C/SGLI, we used a normal distribution with means of typical orders of magnitude and fairly large standard deviations to avoid excessive reliance on the prior distribution. To clarify these points, the following text has been added in the revised manuscript.

[Section 2.2.1; Lines 221 - 225]

"Note that the values in Table 1 could be assigned more appropriate prior distributions (mean, standard deviation, and even covariance) by using cloud property products from other satellite observations. However, since this is the first application of our algorithm to GCOM-C/SGLI, we used a normal distribution for simplicity with means of typical orders of magnitude and fairly large standard deviations to avoid excessive reliance on the prior distribution."

6. *Detail how you estimate the uncertainty in cloud-base height (CBH) from your optimal estimation algorithm. This should include a discussion of the error propagation and any assumptions made in the uncertainty analysis.*

Answer: As the reviewer has pointed out, based on the error propagation theory (employing the optimal estimation framework, the covariance matrix $\boldsymbol{S}_e$, and the forward calculation), it is possible to estimate the uncertainty in the CBH estimation. However, the uncertainty in CBH estimated in this manner is typically underestimated because the primary sources of estimation error arise from factors not accounted for in the forward calculation, such as the vertical inhomogeneity of the cloud property profile and multilayer cloud structures. Therefore, this study assessed the uncertainty in CBH through validation against CBH measurements obtained from ground-based and ship-borne ceilometers (Figures 6 and 9), as well as satellite-borne radar and lidar (Figure 10). In fact, the bias in CTH (Figure 9) and the dependence of CBH bias on multi-layer cloud structure (Figures 6 and 7) observed from these validation analysis are not captured by estimates based solely on the error propagation theory.

7. *Consider performing a sub-analysis by classifying clouds into different types and reporting the results separately. This would help isolate the impacts of cloud type on your findings and provide valuable insights into the variability in estimation performance across cloud types.*

Answer Thank you for your valuable suggestion. We agree that it is important to investigate in more detail how our algorithm behaves for different cloud types. However, we do not have an 'objective information/method' that decomposes the validation results shown in Figures 6, 9, and 10 into different cloud types. For instance, the all-sky camera used to identify cloud types is not necessarily operated at the same time as the ceilometers. Additionally, although observations from CloudSat and CALIPSO are effective for identifying cloud types, they are unfortunately not collocated with SGLI, except in high-latitude regions, because the A-Train satellites operate in the afternoon orbit while GCOM-SGLI operates in the morning orbit. Although it is possible to apply COT and CTH retrieved from SGLI to the traditional cloud type classification of the ISCCP, this classification itself is dependent on the retrieval errors of COT and CTH themselves, and the uncertainty in cloud type classification may not be fully resolved. One potential approach could be to use cloud type classification from geostationary meteorological satellites as an objective third-party. However, such analysis is beyond the scope of this study and we would like to leave it as a topic for future work.

8. *Some new cloud base height retrieving method should be cited in the Introduction. such as: Retrieving cloud base height from passive radiometer observations via a systematic effective cloud water content table, Remote Sensing of Environment, 294 (2023), 113633.*

Answer: Thank you for your valuable suggestion of a literature. We have incorporated it into both the Introduction and References as follows:

[Section 1; Lines 76 - 79]

"Another approach is to estimate other cloud properties correlated with CBH and CGT, such as CTH and COT, which are usually measured using passive instruments not equipped with oxygen absorption channels. This approach has been implemented using adiabatic (Seaman et al., 2017) or statistical models (Noh et al., 2017, 2022; Shao et al., 2023; Tan et al., 2023)."

[References]

"Tan, Z., Ma, S., Liu, C., Teng, S., Letu, H., Zhang, P., and Ai, W.: Retrieving cloud base height from passive radiometer observations via a systematic effective cloud water content table, Remote Sens. Environ., 294, 113633, https://doi.org/10.1016/j.rse.2023.113633, 2023."