First and foremost, we would like to express our sincere gratitude to Luca Lelli, the anonymous reviewers, the editor, and the editorial support team for taking the time to review our manuscript and provide valuable feedback. The comments we received were extremely helpful in improving our manuscript, and we are very grateful for them. As outlined below, we have revised the manuscript based on the feedback. The reviewers' comments are copied below and shown in *italics*, while our responses and the corresponding text in the manuscript are shown in red and orange, respectively.

Response to the editorial support team

Regarding figures 3, 7: please ensure that the colour schemes used in your maps and charts allow readers with colour vision deficiencies to correctly interpret your findings. Please check your figures using the Coblis – Color Blindness Simulator (https://www.color-blindness.com/coblis-color-blindness-simulator/) and revise the colour schemes accordingly with the next file upload request.

Answer: In response to the comment, we updated the color scheme for Figures 3 and 4 (excluding Figure 3a) to the 'Scientific Color Maps' recommended on the AMT submission page (https://www.atmospheric-measurement-techniques.net/submission.html).We recognize that adjusting the color scheme of the RGB images in Figures 3a and 7 as well would also be preferable. However, since the values of the three channels are directly assigned to R, G, and B, we are unsure how to modify them to make them colorblind-friendly. Instead, we utilized the 'Coblis – Color Blindness Simulator' to confirm that the RGB images in Figures 3 and 7 can be correctly interpreted by readers with anomalous trichromacy.

Response to Anonymous Referee #4

The submission by Nagao et al. combines a 4-channel 4-property cloud retrieval with a cloud phase differentiation to get liquid and ice cloud properties output in a single product. This is a neat idea and I found the paper pretty well structured and easy to read. I appreciate the authors found whatever validation data they could and I liked the comparison of cloud base heights (CBH) with reanalysis lifting condensation level.

We would like to thank you very much for carefully reading our manuscript and providing us with valuable comments. We have revised our manuscript, by taking full account of the referee's suggestions. The original comments are copied below and shown in *italics*, while our responses and the corresponding text in the manuscript are shown in red and orange, respectively.

I have a few comments I would request the authors address but I would be happy to support publication following improvements. My comment areas can be summarised as: (1) fixing some issues in the literature review and process description, (2) clarifying some method details, (3) expanding slightly on the verification step, (4) doing some quick theoretical error quantification calculations.
I don't believe my requests would greatly change the results or conclusions, but would improve the quality of the manuscript.

1. LITERATURE REVIEW AND BACKGROUND
Beginning L53 I read it as talking about studies that retrieved BOTH cloud top heights (CTH) and other geometric information (e.g. thickness), but it appears Desmons et al. (2017) gets a single pressure and Davis et al. (2018) talks specifically about how DISCOVR can only get one piece of vertical information. The latter is a two-part analysis and I think both are worth citing, including doi: 10.1016/j.jqsrt.2018.09.006 . Either rephrase, remove these citations, or see my comments a couple of paragraphs below here to see where they could fit.

Answer: We have removed Desmons et al. (2017) and Davis et al. (2018) from the relevant paragraph. However, we believe that these papers, along with the other paper by Davis et al. that you mentioned newly, are significant contributions to the field of remote sensing using oxygen absorption channels. Therefore, we have added a new paragraph immediately after citing these three papers, as follows:

[Section 1; Lines 69 - 75]

"In addition to the literatures cited in the previous paragraph, there are several earlier studies that have investigated or attempted remote sensing of cloud geometric information using oxygen absorption channels while not retrieving both CBH and other geometric information (e.g. thickness). Examples include the use of oxygen A-band measurements (O'Brien and Mitchell, 1992), oxygen B-band measurements from the Global Ozone Monitoring Experiment (Desmons et al., 2019), and two channels in the oxygen A-band and B-band of the Earth Polychromatic Imaging Camera (EPIC) on the Deep Space Climate ObserVatoRy (DSCOVR) (Davis et al., 2018a; 2018b).

"

On L124 you state "Therefore, for a given CTH, the amount of oxygen above the clouds is uniquely determined", but isn't it for cloud-top pressure?

Next up, I would suggest using the path-length framing for parts of your description as you judge appropriate. Lower or thicker clouds = photons travel further and there is more absorption by O2. Then explain how it is hard to work out how much absorption is above versus within cloud, which is why your retrieval relies heavily on another source for CTH. I personally find this framing much simpler and clearer and I think some readers who are not A-band experts will appreciate it.

Answer: Thank you for your thoughtful suggestion. We have revised the first few paragraphs of Section 2.2.1, which explains how the oxygen A-band channel is sensitive to variations in both CTH and CGT, by incorporating your proposal as follows:

[Section 2.2.1; Lines 124 - 147]

"The top-of-atmosphere (TOA) reflectance in the oxygen A-band channel (~763 nm), which is characterized as moderate to strong oxygen absorption, exhibits sensitivity to both CTH and CGT. This mechanism can be described as follows. First, for simplicity, we assume a black surface with an albedo of zero, gas absorption only by oxygen, and a geometrically thin (CGT~0) but optically reasonably thick (COT > 0) plane-parallel cloud. Additionally, we assume that the cloud properties used to parameterize cloud scattering—namely, COT, CER, and cloud thermodynamic phase, are known. Under these conditions, the TOA reflectance in visible and near-infrared channels outside the oxygen absorption band (e.g., the VN9, VN11, and SW1 channels of SGLI, centered at 673 nm, 868 nm, and 1.05 μm, respectively) can be accurately represented using these cloud property parameters.

In the oxygen absorption channel, sunlight is significantly absorbed by the oxygen above

the clouds before and after being reflected by the clouds on its path to the satellite. The TOA reflectivity in the oxygen absorption channel can be expressed with two additional parameters: CTH and the amount of oxygen above CTH. Conveniently, oxygen is well-mixed in the atmosphere, and its mixing ratio can be assumed to be globally constant and known. Thus, if the CTH (or cloud top pressure, equivalently) is given, the amount of oxygen above cloud can be immediately calculated. Therefore, when CGT~0, it is sufficient for parameterizing the TOA reflectance in the oxygen absorption channel to have CTH in addition to COT, CER, and cloud thermodynamic phase.
When CGT > 0, the CGT (or CBH) needs to be included as an additional parameter for explaining the TOA reflectance in the oxygen absorption band. When the cloud is geometrically thickened without changing the CTH (i.e., when CGT increases and CBH decreases), the TOA reflectance should decrease owing to oxygen absorption within the cloud layer between CTH and CBH. This is because, as the CBH decreases, sunlight travels a longer distance in the cloud layer, increasing the opportunities for oxygen absorption. These explanations provide an intuitive understanding of why the TOA reflectance in the oxygen absorption channel is sensitive to variations in both CTH and CGT (or CBH).”

Finally, on L131—135 you mention you would need a couple of channels to separate cloud-top and cloud-thickness, but isn’t that assuming you already know other stuff like optical depth? Actually, I think you’re underselling the difficulty of getting geometric thickness here and readers should know that you’re attempting something that is very challenging. You can cite O’Brien and Mitchell (1992, doi: 10.1175/1520-0450(1992)031<1179:EEFROC>2.0.CO;2) and/or Richardson et al. (2018, doi: 10.5194/amt-11-1515-2018) as those papers show the challenging spectral resolution requirements for pure A-band approaches. The Desmons and Davis references could fit here.

Answer: In response to the comment, we have revised the paragraph containing the sentence you mentioned as follows:
[Section 2.2.1; Lines 148 - 151]
“The retrievals of CTH and CGT from the oxygen absorption band through this principle can be facilitated by a pair of spectral channels with different sensitivities to these cloud parameters since the oxygen absorption channels are sensitive to both CTH and CGT. This is the case even when cloud properties other than CTH and CGT (i.e., COT, CER, and cloud thermodynamic phase) are known.”

1. METHODOLOGY DETAIL

I think you did a nice job here, being efficient in your explanations and I believe I could reproduce a lot of your retrieval thanks to details like Table 1. However, I haven't used RSTAR. Can you specify the gas absorption spectra source, including if/how line broadening is handled. Is there anything else that's relevant: e.g. effective spectral resolution or the angular resolution of your scattering calculations?

Answer: We have added a text providing information about the gas absorption tables included in the RSTAR7 package:

[Section 2.2.2; Lines 234 - 237]

"The RSTAR version 7 (RSTAR7) package includes gas absorption line tables compiled into narrow bands using the k-distribution method. However, the k-distribution table is based on the HITRAN 2004 molecular spectroscopic database (Rothmana et al., 2005) and does not incorporate recent updates to the oxygen absorption lines."

Could you also comment briefly on the cloud flag – how does it handle optically thin or broken clouds? I understand your retrieval assumes plane parallel, but a general statement on whether we should expect lots of broken clouds to be retrieved would help me to interpret things.

Answer: Since the assumption is a plane parallel cloud structure, contamination of clear-sky regions at the sub-pixel scale near the cloud edge causes significant bias in cloud retrievals (Zhang and Platnick, 2011) In scenes with small cumulus clouds, which are prone to clear-sky contamination, quality flags based on indicators such as the spatial standard deviation of TOA reflectance in the VNIR channels or the retrieved COT would be useful. Although such flags have not been implemented in this study, it would be valuable to carry out such flagging if the target scene can be identified as primarily consisting of cumulus clouds.

- Zhang, Z. and Platnick, S.: An assessment of differences between cloud effective particle radius retrievals for marine water clouds from three MODIS spectral bands, Journal of Geophysical Research: Atmospheres (1984–2012), 116, https://doi.org/10.1029/2011jd016216, 2011.

1. VALIDATION

When you compare against the ground-based ceilometers you use the local standard deviation and "empirically determined" some data selection. I don't think we can be sure that the result is appropriate to compare with your values estimated from the previous section, because you might have filtered for "best behaving" scenes. Is there a way to identify these "better" scenes from your satellite product? If not then you should explicitly state that this as a potential issue for the comparison.

Answer: As the reviewer pointed out, only SGLI scenes with a local standard deviation of ceilometer CBH below 1 km were used for validation in Figure 6 of Section 4.2. We believe this data screening is essential for the following reasons. First, since ceilometer cannot perform spatially wide measurements, ensuring temporal stability would allow us to extract a spatially homogeneous cloud field. Additionally, this screening helps minimize the impact of local variability in ceilometer CBH on the RMSE of SGLI CBH relative to ceilometer CBH.

However, as you commented, this screening may inadvertently result in selecting scenes and cloud types that are more favorable for SGLI cloud retrievals. This is because factors that cause significant biases in cloud retrievals, such as sub-pixel scale heterogeneity of COT and CBH, are more likely to be common in scenes with diverse cloud types, such as cumulus and cirrus clouds, which are filtered out by this screening. Although it may be possible to identify these factors using indicators such as the spatial standard deviation of cloud optical thickness, this study does not implement such quality flags. To clarify this concern, we have revised the main text as follows:

[Section 4.2; Lines 443 - 451]

"This is because data with significant temporal variability are more likely to encompass diverse cloud types, such as cumulus and cirrus clouds. Notably, these three thresholds (Δs < 4 km, Δt < 30 min, and $\mathrm{SD}[CBH_{ground-based}] <$ 1 km) were empirically determined. However, it is important to note that the data screening using this $\mathrm{SD}[CBH_{ground-based}] <$ 1 km might unintentionally exclude scenes and cloud types that are challenging to retrieve. This is because factors that cause significant biases in cloud retrievals, such as sub-pixel scale heterogeneity of COT and CBH, are more likely to be common in scenes with diverse cloud types, such as cumulus and cirrus clouds, which are filtered out by this screening. Therefore, it should be noted that the SGLI cloud properties retrieved using our algorithm may contain lower-quality data than those presented in the validation results here."

Can you do the CloudSat comparison split by phase? It would be cool to see, but it's not worth much extra effort on your part so I am not pushing hard for this. If you have to reprocess all the granules to get your fields then don't bother - it's only a minor request.

Answer: Thank you for the valuable suggestion. It is possible to classify CloudSat/CALIPSO-derived CTH/CBH using the cloud thermodynamic phase information identified from CALIPSO lidar measurements, included in the CloudSat 2B-CLDCLASS-LIDAR product. However, because the footprint of CALIPSO lidar and the swath of SGLI rarely overlap, except in high-latitude regions (CloudSat and CALIPSO operate in the afternoon orbit, while GCOM-C/SGLI operate in the morning orbit), the CALIPSO-derived cloud phase cannot be used for the classification of the SGLI CTH/CBH. Our algorithm also retrieves cloud phase information (ice COT fraction as shown in Figures 3d and 4c) utilizing the SWIR channels of SGLI, but it would capture the cloud phase for deeper optical depths compared to CALIOP, which is limited to detecting the cloud phase near the cloud tops (Nagao and Suzuki, 2022 JGR). As a third source capable of uniformly classifying CTH/CBH from both CloudSat/CALIPSO and SGLI, the use of cloud phase obtained from geostationary satellites, such as Himawari and GOES series, could be considered. However, such analysis is beyond the scope of this study and we would like to leave this as a subject for future work.

- Nagao, T. M. and Suzuki, K.: Characterizing Vertical Stratification of the Cloud Thermodynamic Phase With a Combined Use of CALIPSO Lidar and MODIS SWIR Measurements, J Geophys Res Atmospheres, 127, https://doi.org/10.1029/2022jd036826, 2022.

1. UNCERTAINTY TESTS

We know that cloud properties correlate, but I'm fine with you sticking with a diagonal prior given the data we have available. Also, we know that the forward model also has error, which it seems you don't include in your optimisation. Therefore your retrieval might be too "tight" to the observations, right? I think these are legitimate concerns that would take a full other project to appropriately quantify. However, you could address them by re-running a subset of the retrieval footprints and evaluating how the retrievals *change. I request 3 tests*

Answer: Thank you for your valuable comment. We agree that in order to run the optimal estimation method most effectively, it is important to assign appropriate parameters, including correlations between cloud properties, to a prior distribution of the state vector ($\boldsymbol{S}_a$). We also understand that correlations between cloud properties, such as COT, CTH, and CBH, which are derived from satellite observations, can be utilized in $\boldsymbol{S}_a$. However, this paper represents one of the first applications of the GCOM-C SGLI oxygen A-band channel (VN9), and its primary objective is to demonstrate that CGT and CBH information can be extracted from SGLI VN9 measurements. Therefore, we assigned relatively loose prior distributions to $\boldsymbol{S}_a$, as shown in Table 1, and designed the algorithm to be tightly constrained by the SGLI measurements. While utilizing correlations between cloud properties derived from satellite observations in the optimal estimation method is a promising approach, we consider it as a challenge for future work.

- *Try with different priors to see whether your results are meaningfully sensitive to them. Shift means term-by-term, and/or scale the standard deviations. The most interesting would be to introduce a cross correlation e.g. between tau and CGT/CBH. Even if it's only a small correlation*

Answer: As mentioned above, assuming correlations between cloud properties (i.e., modifying the off-diagonal elements of $\boldsymbol{S}_a$) does not align with the purpose of this paper. Therefore, we have decided not to include it here. Instead, we tested the algorithm using a perturbed $\boldsymbol{x}_a$ and relaxed $\boldsymbol{S}_a$, and the results are presented in Figures S6 and S7, respectively, of the revised supplemental material. Figure S6 compares two cloud retrievals: the x-axis represents the original retrieval under the default settings described in the main text, while the y-axis represents the retrieval obtained when the elements of $\boldsymbol{x}_a$ were assigned random numbers following a uniform distribution within the ranges defined in Table 1. Figure S7 is similar to Figure S6, except the y-axis represents the retrieval obtained under a relaxed priori distribution of $2\boldsymbol{S}_a$. These figures demonstrate that perturbations to $\boldsymbol{x}_a$ and relaxations of $\boldsymbol{S}_a$ primarily increase the RMSEs of retrieved cloud properties, while not having a significant impact on the biases of them. Therefore, the following text has been added to the main text as well:

[Last paragraph of Section 5.2; Lines 691 - 695]

"Moreover, the series of analyses shown in Figures S6 - S8 illustrate that our algorithm can be influenced by a priori information regarding the state vector and the measurement vector, particularly the settings of $\boldsymbol{x}_a$, $\boldsymbol{S}_a$, and $\boldsymbol{S}_e$. It remains a matter for future research to investigate how to objectively determine the optimal settings for these a priori parameters."

- *A subset of simulations with* $\mathbf{y}$ *elements perturbed in turn? The perturbation magnitudes should probably be proportional to the standard deviations implied by S_e for consistency. This would tell us something about how errors in each particular channel would feed through into the overall retrieval.*

Answer: In the supplemental material, we have included a section (Text S2) on sensitivity analysis based on error propagation theory and radiative transfer simulation with their results shown in Figures S1 and S2. We consider Figure S1 to be a direct response to this comment. Specifically, Figure S1 demonstrates how perturbations of signals in SW1, SW4, SW3, TI1, and VN9 propagate to the retrievals of COT, CER, ICOTF, CTH, and CBH. The horizontal panels in Figure S1 correspond to the measurement errors applied to specific channels, while the vertical panels correspond to the impact of these errors on retrievals of cloud properties. For example, Figure S1 (1-5,1) shows the COT error caused by the perturbation in SW1. One important takeaway from Figure S1 is that the perturbation in SW1 has a relatively large impact not only on COT but also on CBH. This suggests that the accuracy of COT is crucial for extracting CBH information from VN9. Therefore, the following text have been added to the main text as well:
[Last paragraph of Section 4.1; Lines 412 - 425]
"In our algorithm, the uncertainty in CBH retrieval is also entangled with the uncertainty in COT retrieval. We performed a sensitivity analysis based on the error propagation theory to examine how measurement uncertainties propagate to retrieval uncertainties (see Text S2 in the supplemental material). Figure S1 demonstrates how perturbations in individual measurement channels induce retrieval errors. Notably, perturbations in SW1, which is a channel sensitive to COT but located outside the oxygen A-band, can induce errors not only in COT retrieval (Fig. S1(1,1)) but also in CBH retrieval (Fig. S1(5,1)). This indicates that COT errors disturb the separation of COT and CBH from VN9 measurements. Figure S2 further demonstrates how the overall uncertainty in the multi-wavelength measurements incorporated into the inverse estimation propagates to retrieval uncertainties. The comparison of Figs. S2a1 and S2b1 reveals incorporating VN11 alongside SW1 reduce the uncertainty in COT retrieval, which, in turn, contributes to

reduce uncertainty in CBH retrieval. As described in Section 2.2.1, our algorithm utilized both SW1 and VN11. The results of these sensitivity analyses emphasize the importance of carefully addressing uncertainties in COT retrieval when deriving CBH from VN9 measurements. The entanglement of COT, CTH, and CBH retrieval errors associated with oxygen A-band measurements has also been reported by Lelli et al. (2014)."

- *Some tests with a relaxed S_e, just scale it to be larger. This would represent the effect of model error.*

Answer: We tested the algorithm using a relaxed $\boldsymbol{S_e}$, with the results presented in Figures S8 of the supplemental material. Figure S8 compares two cloud retrievals: the x-axis represents the original retrieval under the default settings described in the main text, while the y-axis represents the retrieval obtained under an increased measurement uncertainty of $2\boldsymbol{S_e}$. Figure S8 shows that COT and CER tend to be underestimated for clouds with large values of COT and CER. This is believed to occur because as COT and CER increase, the sensitivity decreases (with smaller changes in TOA reflectance), and as $\boldsymbol{S_e}$ increases, the iteration toward the solution from the initial values tends to converge earlier. However, this underestimation of COT and CER does not have a significant impact on ICOTF, CTH, and CBH.
Therefore, the following text has been added to the main text as well:
[Last paragraph of Section 5.2; Lines 691 - 695]
"Moreover, the series of analyses shown in Figures S6 - S8 illustrate that our algorithm can be influenced by a priori information regarding the state vector and the measurement vector, particularly the settings of $\boldsymbol{x_a}, \boldsymbol{S_a}$, and $\boldsymbol{S_e}$. It remains a matter for future research to investigate how to objectively determine the optimal settings for these a priori parameters."

MINOR COMMENTS:
L280 – why the change in resolution? Why specified dates?

Answer: Unfortunately, our algorithm is not fast enough to process the original SGLI LTOAK radiance product, which is projected onto a sinusoidal tile grid at a resolution of 1/120° (~1 km) and used in Figure 3, across the entire globe for a single year. To demonstrate an example of global processing shown in Figure 4, we used the SGLI

LTOAF product—an alternative radiance dataset provided by the GCOM-C mission, created by downsampling the LTOAK files to a resolution of 1/24°.

L319—327: "it is not surprising" and associated text seems like a long winded way of saying that a cloud with 3 km cloud top cannot be 6 km thick… you could cut this down for brevity.

Answer: In response to the comment, the text in the corresponding paragraph has been revised and shortened as follows:

[Section 3.2; Lines 363 - 366]

"The global distribution of CGT in Fig. 4f also agrees with typical climatological cloud regimes and was similar to that of CTH. For example, mean CGTs of less than 2 km were observed in cumulus cloud regions within the trade-wind zone and in stratocumulus decks over the ocean off the western coasts of continents, whereas mean CGTs greater than 6 km corresponded to deep convective clouds in the tropics."

L407: "SGLI, VN3, SW3, and SW4 channels" I think the first comma should be removed to read "SGLI VN3, SW3, and SW4 channels". The current writing makes it seem like SGLI is part of the list.

Answer: We have removed "," after SGLI.

L481—484: this bit just seems obvious. You could lose the last two sentences.

Answer: In response to the comment, the text in the corresponding paragraph has been revised and shortened as follows:

[Section 4.4; Lines 547 - 550]

"This section presents a comprehensive validation of both CTH and CBH derived from SGLI, complementing the validation of CBH alone presented in the previous two sections. As illustrated in Fig. 2, the accuracy of CBH retrieval using the 763 nm channel is shown to depend on the accuracy of CTH retrieval. Therefore, in addition to the CBH-only validation, a thorough validation of both CTH and CBH retrievals is necessary."

L519—520: "One is the presence of optically thin cirrus clouds overlying opaque clouds, *which can only be detected by CALIOP. In such cloud vertical structures, the CTH and CBH retrieved by our algorithm are expected to correspond to those of the opaque clouds" – won't this be a bit more complicated and you might end up with something in between? How far into the clouds does the TIR channel typically see?*

Answer: Typically, TIR can retrieve cirrus clouds with a COT of 0.1 or greater (e.g., Iwabuchi et al., 2016), whereas the CALIOP lidar can detect optically thinner cloud layers with optical thicknesses as small as about 0.01 (e.g., Winker et al., 2009). Notably, CALIOP observations have revealed that cirrus clouds with COT $<$ 0.1 are relatively common. Based on these facts, we assumed the existence of vertical cloud layers detectable by CALIOP alone but transmissive to TIR radiation. To clarify these points, the text has been revised as below.

[Section 4.4; Lines 584 - 589]

"One is the presence of optically thin cirrus clouds overlying opaque clouds, which can be detected by CALIOP but are transmissive to TIR radiation. This arises because TIR typically retrieves cirrus clouds with COTs of 0.1 or greater (e.g., Iwabuchi et al., 2016), while the CALIOP lidar is capable of detecting optically thinner cloud layers with optical thicknesses as small as about 0.01 (e.g., Winker et al., 2009). In such vertical cloud structures, the CTH and CBH retrieved by our algorithm are expected to correspond to those of the opaque clouds"

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