



Aircraft Evaluation of MODIS Cloud Drop Number Concentration Retrievals

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Abstract. Cloud droplet number concentration (N_d) can be retrieved through passive satellite observation. These retrievals are useful due to their wide spatial and temporal coverage. However, the accuracy of the retrieved values is not well understood. In this paper, we compare satellite N_d derived from the Moderate Resolution Imaging Spectroradiometer (MODIS) instrument with in situ aircraft measurements using a phase Doppler interferometer onboard three flight campaigns sampling marine stratocumulus clouds. Intercomparison of N_d values shows that the discrepancy between retrieved and in situ N_d can be $\pm 50\%$ or more. In the mean, there is evidence of an overestimation bias by MODIS retrievals, although the sample size is insufficient for statistical certainty. We find that MODIS N_d is best interpreted as representative of the mid-cloud region, as there is almost always a greater discrepancy from in situ values near cloud top and cloud base. We also find evidence of cases where N_d is accurately retrieved, but effective radius is not, presumably due to offsetting errors in other retrieval parameters. Vertical profiles of extinction coefficient β , liquid water content L , and effective radius r_e measured during sawtooth-pattern flight legs through cloud top are also compared to implicit MODIS retrieval profiles. For the one case with N_d agreement, all profiles match well. For seven cases with significant disagreement, there is no consistent underlying cause. The discrepancy originates from either: (a) discrepancy in the r_e profile, (b) discrepancy in the β and L profiles, or (c) discrepancy in both.

1 Introduction

Cloud drop number concentration (N_d) is a fundamental property of clouds. This quantity is relevant to cloud properties that impact weather and climate, such as cloud radiative effects, precipitation, and aerosol-cloud interactions. Passive satellite observation is one of the primary methods that we have to measure and understand the climatology of N_d at large spatial and temporal scales. The Moderate Resolution Imaging Spectroradiometer (MODIS) instrument onboard the Terra and Aqua satellites is one important source of such observations. However, the estimation of N_d relies on assumptions which may or may not portray cloud properties accurately in all situations. The focus of this study is to understand and compare MODIS retrievals against in situ measurements to evaluate the accuracy of the satellite N_d product. In addition, we seek to identify the underlying cause of any discrepancies observed between MODIS and in situ observations. When we find good agreement, is it because the underlying properties are also in agreement, or are there cases of compensating errors? When we find poor agreement, is there one consistent reason for it, or is there a diversity of reasons? A review paper by Grosvenor et al. (2018)



25 comprehensively surveys our understanding of deriving N_d from remote sensing, so we focus this Introduction only on aspects that are directly relevant for this study.

1.1 MODIS N_d Retrieval

The most common MODIS retrieval of number concentration utilizes the following equation (Grosvenor et al., 2018):

$$N_d = \frac{\sqrt{5}}{2\pi k} \left(\frac{f_{ad} c_w \tau_c}{Q_{ext} \rho_w r_e^5} \right)^{1/2} \quad (1)$$

30 Cloud optical depth τ_c and cloud top effective radius r_e are the two (mostly) independently retrieved quantities in the MODIS N_d retrieval. Retrieved N_d is more sensitive to the same relative uncertainty in retrieved r_e than τ_c due to the difference in the magnitude of the exponents (5/2 versus 1/2) in Eq. 1. Density of water ρ_w is known, and the remaining variables are considered fixed as described below. In order to estimate r_e , MODIS makes use of a weighting function, defined as (Platnick, 2000):

$$W(\tau_c) = a\tau_c^b \exp\left(-\tau_c\left(\frac{1}{\mu} + \frac{1}{\mu_0}\right)\right) \quad (2)$$

35 Here, $b = 2$, a is a normalization constant, and μ and μ_0 depend on satellite position and correspond to the solar zenith angle and sensor zenith angle, respectively. The weighting function describes, as a function of cloud optical depth, how much influence the measurement from a given region of a cloud has on satellite-derived variables. This function peaks within a few tens of meters of cloud top, and therefore the effective radius reflects values in this cloud top region.

Cloud optical depth τ_c is defined as the vertical integral of the extinction coefficient $\beta(z)$:

$$40 \quad \tau_c = \int_{z_{base}}^{z_{top}} \beta(z) dz \quad (3)$$

where z_{top} and z_{base} are the altitude of cloud top and cloud base, respectively.

At any altitude, $\beta(z)$ is related to the cloud drop size distribution $n(r)$:

$$\beta(z) = \int_0^\infty \pi Q_{ext}(r) n(r, z) r^2 dr \quad (4)$$

The remaining variables in the N_d retrieval (Eq. 1) are described as follows:

45 1. N_d is assumed to be a constant with respect to height in the cloud, i.e. $N_d(z) = constant$.

2. k is defined as:

$$k = \left(\frac{r_v}{r_e} \right)^3 \quad (5)$$



where r_v is volume-mean droplet radius. The MODIS retrieval assumes that $k = 0.8$. Previous studies suggest that k is well-constrained in stratocumulus clouds, typically ranging between 0.7 and 0.9 (Miles et al., 2000; Lebsack and Witte, 2023).

3. Q_{ext} is the extinction efficiency (dimensionless), and represents the ratio between the extinction and physical cross sections of a droplet. Because the radius of the droplets of interest is usually much larger than the wavelengths of light used for retrievals, MODIS assumes $Q_{ext} = 2$ (the limit for geometric optics) (Platnick, 2000).

4. c_w is the adiabatic gradient of liquid water content L with respect to height, and is a weak function of temperature and pressure. From Eq. 14 in Grosvenor et al. (2018), we compute a value of $c_w = 2.3 \times 10^{-6} \text{ kg/m}^4$. We assume c_w to be constant vertically through a cloud, which should introduce an error that is less than 1% since the stratocumulus observed in this study are quite geometrically thin ($< 500 \text{ m}$) (Grosvenor et al., 2018).

5. f_{ad} is defined as the fraction of cloud liquid water content relative to its adiabatic value at a given height above cloud base. The MODIS retrieval assumes $f_{ad} = 0.6$. Combining the definitions of c_w and f_{ad} yields the profile of liquid water content:

$$L(z) = f_{ad}c_w(z - z_{base}) \quad (6)$$

1.1.1 Retrieval profiles

The MODIS retrieval implicitly assumes specific vertical profiles of r_e , β , and L . These profiles, along with cloud base height, can be derived starting with measured r_e and inferred N_d as follows:

– The cloud top liquid water content is computed as:

$$L(z_{top}) = \frac{4}{3}\pi\rho_w \cdot kr_e^3 \cdot N_d \quad (7)$$

– The liquid water content profile $L(z)$ is defined above (Eq. 6).

– Cloud base height z_{base} is determined by the altitude where $L(z) = 0$. By re-arranging Eq. 6 and applying it at $z = z_{top}$, we get:

$$z_{base} = z_{top} - L(z_{top})/f_{ad}c_w \quad (8)$$

– $r_e(z)$ is derived starting from the definition of r_v :

$$\frac{4}{3}\pi r_v^3(z)N_d\rho_w = \frac{4}{3}\pi [kr_e^3(z)]N_d\rho_w = L(z)$$



which can be re-arranged in terms of r_e :

$$r_e(z) = \left[\frac{3}{4\pi\rho_w k} \frac{L(z)}{N_d} \right]^{1/3} \quad (9)$$

75 – $\beta(z)$ is derived by substituting the definitions of r_e and L into Eq. 4, which yields (Grosvenor et al., 2018):

$$\beta(z) = \frac{3}{4} \frac{Q_{ext}}{\rho_w} \frac{L(z)}{r_e(z)} \quad (10)$$

In order to better identify the source of any discrepancies between MODIS and in situ N_d , cloud base height z_{base} , along with the vertical profiles $r_e(z)$, $\beta(z)$, and $L(z)$ that are inherent in the MODIS algorithm for estimating N_d will be compared to in situ observations of the same quantities. There are a number of potential sources of uncertainty, including the MODIS
80 retrievals of r_e and τ_c , as well as the validity of the above assumptions. Grosvenor et al. (2018) estimates the overall uncertainty in retrieved N_d to be $\pm 25\%$.

2 Methods

2.1 In Situ Observations of N_d

2.1.1 Aircraft Observations

85 In situ data was acquired during three different flight campaigns that sampled marine stratocumulus: the Marine Stratus/Stratocumulus Experiment (MASE; Lu et al., 2007), the Physics of Stratocumulus Top experiment (POST; Carman et al., 2012; Gerber et al., 2013), and the Variability of American Monsoon Systems Ocean-Cloud-Atmosphere-Land Study (VOCALS; Mechoso et al., 2014; Zheng et al., 2011). Each campaign was flown on the CIRPAS Twin Otter (TO) aircraft with the same phase Doppler interferometer (PDI) that we use to derive cloud microphysical properties. See Chuang et al. (2008) for details
90 about the PDI measurement method and data processing.

The MASE campaign was flown in the NE Pacific near Monterey, California during July 2005. VOCALS was centered off the coast of Chile in the SE Pacific and was flown during October to November of 2008. The flight pattern of both the MASE and VOCALS campaigns was over level legs which sampled from below cloud base to near cloud top. POST was flown slightly farther offshore in a similar location as MASE, during July and August of 2008. Flight legs used from the POST campaign
95 were flown in a sawtooth pattern, flying repeatedly up and down between ~ 100 m below cloud top to ~ 100 m above. Overall, we analyze four flight days from MASE, ten from VOCALS, and eight from the POST campaign, as these coincide with a MODIS overpass. More details on matching the aircraft flights with MODIS overpasses can be found in Witte et al. (2018).

The PDI measurements from these three field campaigns have been used to analyze the retrieval of cloud effective radius from MODIS. Previous studies (Painemal and Zuidema, 2011; Min et al., 2012; Noble and Hudson, 2015) have suggested that
100 MODIS retrievals of r_e are biased high by 2 to 5 μm relative to in situ measurements, but using the same data set as in this



study, Witte et al. (2018) found no such bias, with a discrepancy between MODIS and in situ measurements that agrees, in the mean, to within $0.7 \mu\text{m}$. The bias is instead attributed to issues with the in situ aircraft instrumentation used. This suggests that, in the mean, there is no bias in retrievals of N_d due to a bias in retrieved r_e , an assertion that we will evaluate as part of our analysis in this study.

105 2.1.2 In Situ N_d and r_e Calculations

To estimate cloud drop number concentration from PDI, in-cloud sampling legs are chosen for each flight analyzed. To be consistent with MODIS, cloud top data is used to derive r_e . In contrast, because the satellite retrieval assumes N_d is constant throughout the cloud, mid-cloud data is used to derive number concentration, as this location is the most representative of the cloud as a whole (as will be discussed further below). Cloud top number concentration can be lower than mid-cloud values due
110 to entrainment.

For the VOCALS and MASE campaigns, mid-cloud and cloud top legs are selected during flight by the flight scientist. However, the POST campaign primarily used a sawtooth flight pattern. Therefore, we define cloud top as the altitude where liquid water content crosses a threshold of $L = 0.05 \text{ g/m}^3$. Effective radius is calculated from within 10 m of this cloud top. Mid-cloud is defined by altitudes between 60 m to 90 m below cloud top and used to calculate number concentration.

115 For ease in comparing MODIS cloud profile estimations to observation, we create a shifted altitude (z_{shift}) for POST which is defined by $z_{shift} = 0$ at cloud top, i.e. $z_{shift} = z - z_{top}$. To perform this coordinate transformation, we determine the altitude of cloud top (z_{top}) for each individual ascent or descent (or “leg”) of the sawtooth flight path (using the threshold $L = 0.05 \text{ g/m}^3$). All in situ measurements for each leg are referenced to z_{shift} for that leg. More details about the altitude shifting process can be found in Carman et al. (2012).

120 For the purpose of making a more like-to-like comparison between PDI and MODIS, number concentration and effective radius measured along their respective legs are averaged over 1 km (20 sec) intervals, matching the MODIS spatial resolution. The mean and variability of N_d and r_e for each flight are calculated from these 1 km average values.

2.1.3 Profile Calculations

To estimate N_d , MODIS combines measurements of τ_c and cloud top r_e with assumptions about the cloud vertical structure
125 (see Section 1.1). Therefore, MODIS implicitly assumes specific vertical profiles of effective radius, liquid water content, and extinction coefficient. Due to the sawtooth sampling strategy during the POST campaign, we can compare these assumed profiles with observations. We also evaluate the assumptions that k and f_{ad} are constant.

Profiles of L , r_e , and β are derived directly from cloud drop size distribution measured by the PDI and binned over 5 m increments within z_{shift} space. Consistent with the MODIS scheme, we also assume $Q_{ext} = 2$ when calculating β . To determine
130 $L_{ad}(z)$, first the mean altitude for near-cloud base is identified using a threshold liquid water content of $L = 0.1 \text{ g/m}^3$. Next, c_w is used to extrapolate downward to $L = 0 \text{ g/m}^3$ associated with the true cloud base z_{base} .



2.2 Satellite retrieval sampling methodology

In order to determine MODIS number concentration, 1 km retrievals of r_e and τ_c are taken from satellite swaths that match both the temporal and spatial location of the PDI flight campaigns (for more details on temporal matching, see Witte et al., 2018). We then average these measurements over 25 km² (5 × 5 pixels) in order to compare with in situ measurements. MODIS N_d is then estimated using Eq. 1 as described in Section 1.1.

2.3 Uncertainty calculations

Uncertainty of any variable x is expressed as the standard error:

$$error = \frac{\sigma_x}{\sqrt{n_x}} * z(0.95) \quad (11)$$

where σ_x is the standard deviation of x , and n_x represents the total number of measurements. The quantity $z(0.95)$ is the z-score for the 95% confidence interval and has a value of 1.96.

The MODIS uncertainty is calculated from 1 km measurements over a 25 km² area. PDI uncertainty is calculated from downsampled 1 km measurements over the relevant time interval of a given flight leg. Both the MODIS and PDI uncertainties are a reflection of spatial variability at 1 km. This measurement does not reflect instrumental uncertainty or error in assumptions.

3 Results

We first analyze N_d and r_e for all three campaigns. After that, we capitalize on the sawtooth legs from the POST flights to compare profiles of k , f_{ad} , r_e , β , and L .

3.1 Comparison between MODIS and PDI N_d

Fig. 1 compares MODIS and PDI N_d for all three flight campaigns. Though the majority of flights agree to within 25% (accounting for variability), there are several cases that exceed this range. Of those cases, most are overestimates by MODIS, with the one exception being VOCALS day 2008/11/01 (teal diamond in Fig. 1). At a population level, linear regression yields a slope of 1.1 ± 0.14 (95% confidence interval). Although the one-to-one line is included in this confidence interval, there is a suggestion of an overestimation bias. However, because of the limited sample size it is unclear if this is statistically significant.

There are several questions which arise from this result. First, when MODIS and PDI number concentration are in very good agreement, is the satellite retrieved N_d correct for the right underlying reasons or are there multiple errors which offset each other? Second, when MODIS disagrees with PDI, which MODIS retrieval products and/or assumptions are responsible for the discrepancy? To investigate these questions, we further analyze and compare the underlying variables in the MODIS N_d retrieval.

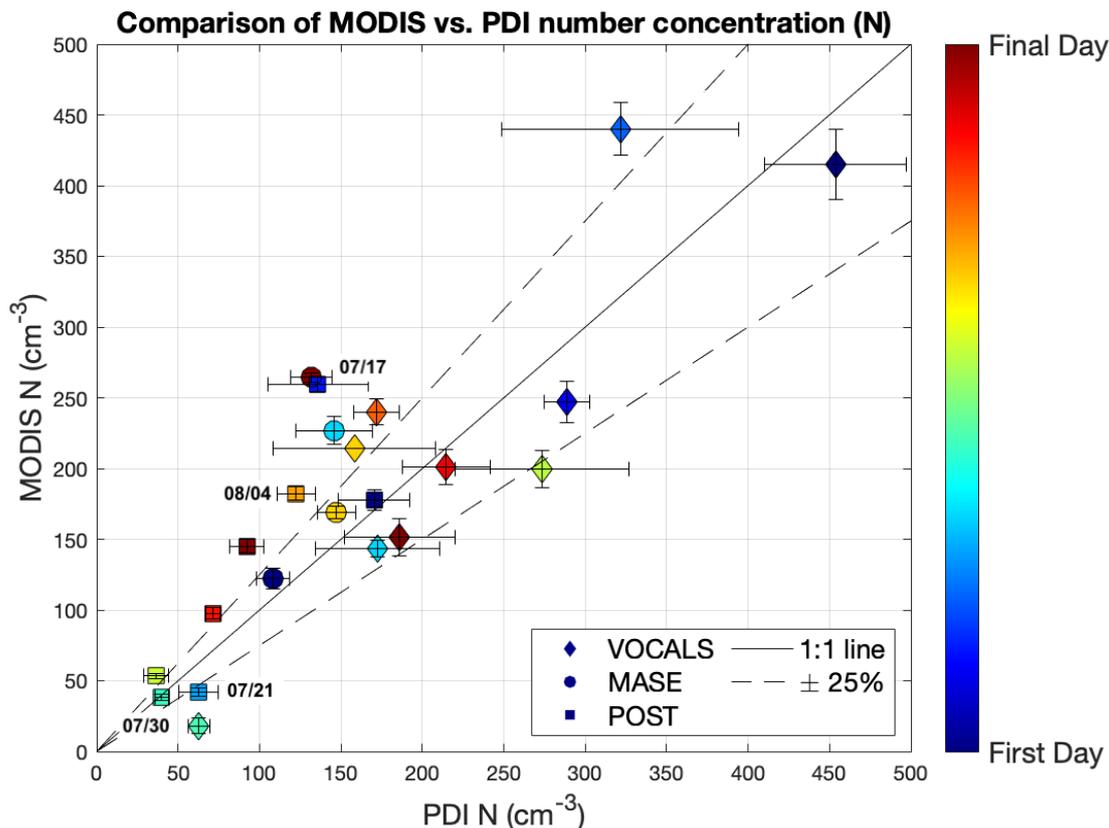


Figure 1. Comparison of MODIS number concentration vs PDI number concentration. PDI values are N_d averaged at mid-cloud for all flight legs on that day, and the uncertainty bar represents 1σ variability of 1-km averaged values. Each flight campaign is marked by a symbol with color ranging from the first day of the campaign to the last. Several days are labeled which represent specific cases discussed in Section 3.3. The MODIS N_d value is a 1 km average of a 5×5 km² swath (with 1σ variability uncertainty bars) using Eq. 1 where effective radius and optical depth are taken from the satellite products. A linear fit of the data produces a slope of 1.1 ± 0.14 .

3.2 Comparison between MODIS and PDI $r_e^{5/2}$

160 Effective radius is the most influential term in the N_d calculation, and thus we speculate that good agreement between satellite and in situ r_e values would likely manifest as good agreement between PDI and MODIS number concentration. Because MODIS $N_d \propto r_e^{-5/2}$ (Eq. 1), Fig. 2 compares MODIS and in situ $r_e^{5/2}$ values across the three campaigns.

Fig. 2 shows that there is a range of agreement between PDI and MODIS, with most of the days agreeing to within 25%. A linear regression of the data produces a slope of 1.0 ± 0.07 (95% confidence interval). This is a result consistent with Witte
 165 et al. (2018) who found no significant bias between MODIS and PDI measured r_e . Additionally, if r_e were the determining factor in the accuracy of MODIS N_d retrievals, it would be expected that the potential high bias of MODIS N_d (Fig. 1) would correspond to a low bias in MODIS r_e . The data does not support this hypothesis, implying that the problem with the satellite estimation cannot solely be attributed to effective radius.

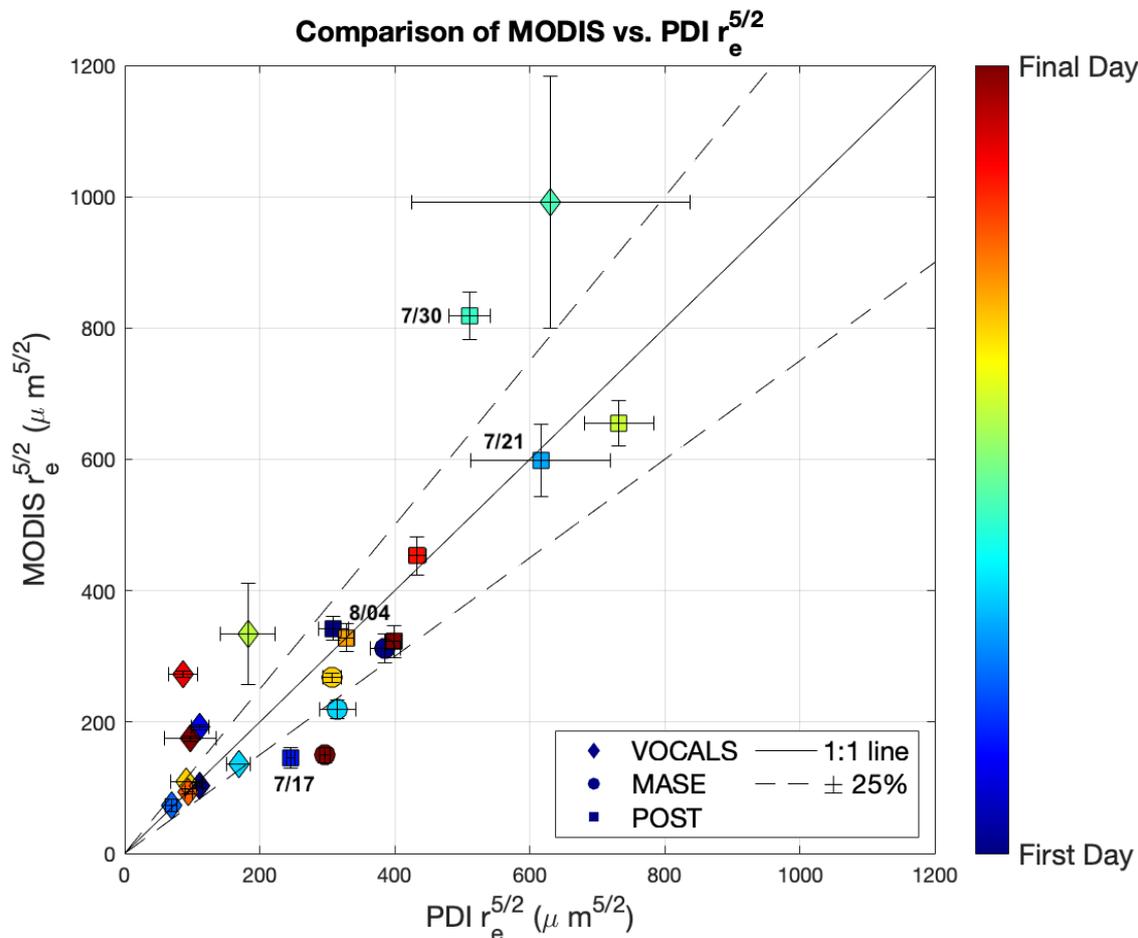


Figure 2. Comparison of $r_e^{5/2}$ between PDI measurements and the MODIS retrieved values. Satellite effective radius is the average of $5 \times 5 \text{ km}^2$ swaths of the MODIS r_e product. The PDI value is r_e averaged for all cloud top flight legs on that day. A linear fit of the data produces a slope of 1.0 ± 0.07 . Labels and uncertainty bars are the same as in Fig. 1.

On an individual flight basis, comparing Fig. 2 and Fig. 1 shows that agreement in N_d does not necessarily equate to agreement in $r_e^{5/2}$ and vice versa. In fact, there are four different scenarios which occur:

- Agreement in $r_e^{5/2}$ with agreement in N_d .
- Agreement in $r_e^{5/2}$ with disagreement in N_d .
- Disagreement in $r_e^{5/2}$ with agreement in N_d .
- Disagreement in $r_e^{5/2}$ with disagreement in N_d .

We illustrate one example of each case in Table 1. The finding that the r_e retrieval does not govern agreement of MODIS N_d leads us to further investigate the other variables in the MODIS N_d calculation.



Good N_d Agreement					
	Symbol	Campaign	Date	%Discrepancy r_e	%Discrepancy N_d
Good r_e Agreement		POST	2008/07/16	-6.8%	+4.1%
Poor r_e Agreement		VOCALS	2008/11/10	+37.0%	+4.0%
Poor N_d Agreement					
	Symbol	Campaign	Date	%Discrepancy r_e	%Discrepancy N_d
Good r_e Agreement		POST	2008/08/04	-0.04%	+33%
Poor r_e Agreement		VOCALS	2008/11/01	+17%	-250%

Table 1. Examples of each of the four possible combinations of N_d and r_e agreement with their associated campaign, day, discrepancy, and corresponding symbol from Figs. 1 and 2. The finding that at least one case has a large discrepancy (37%) in effective radius, with a small discrepancy (4%) in N_d implies that there are compensating errors in other retrieval parameters. In other words, it is possible to retrieve the right N_d for the wrong underlying reasons.

3.3 Specific POST cases

The MODIS calculation for number concentration implicitly relies on vertical profiles of L (through k and f_{ad}), r_e , and β (through τ_c). These profiles combine MODIS measurements of τ_c and r_e with assumptions of cloud vertical structure (see Section 1.1). We compare these profiles with those measured by aircraft during the POST campaign. The sawtooth flight plan of POST allows for detailed profile comparisons. Five POST cases are selected to illustrate the range of behaviors observed across all eight POST flights analyzed in this manner. Fig. 3 illustrates the amount of data for four of the cases. Most of the data is concentrated within 150 m of cloud top. It is difficult to make statistically robust conclusions at altitudes below this region. Within these eight cases, we find one with excellent agreement between all observed and retrieved properties (Case 1). The other seven are cases where N_d is not accurately retrieved. Among these cases, none represents an accurate retrieval of N_d with significant errors in the underlying retrieval properties that compensate for each other, although such cases do exist (see Table 1).

3.3.1 Case 1: POST day 2008/07/30

Fig. 4 is an example of the best case scenario for agreement between MODIS and in situ profiles. The PDI observed number concentration (open circles) calculated at mid-cloud is $N_d = 40 \pm 2 \text{ cm}^{-3}$, while MODIS estimates $N_d = 38 \pm 4 \text{ cm}^{-3}$ (solid red line). The satellite assumes that this number concentration is fixed throughout the vertical cloud profile. In reality N_d is not constant with height, but instead gradually drops off towards cloud base and cloud top. We attribute lower values of N_d near cloud top to cloud drop evaporation due to entrainment. Low values near cloud base may be an effect of uneven cloud base. Through the middle of the cloud N_d does appear reasonably constant on average. Because constant N_d is consistent with the MODIS assumption, we use this as justification for the use of mid-cloud level legs for the MODIS-PDI N_d comparison.

The vertical profiles of L , r_e , and β all agree very closely during this flight. The MODIS estimate of cloud top r_e (blue star) is plotted at an altitude determined by the weighting function (Eq. 2) and is consistent with aircraft measurements. Lastly, cloud

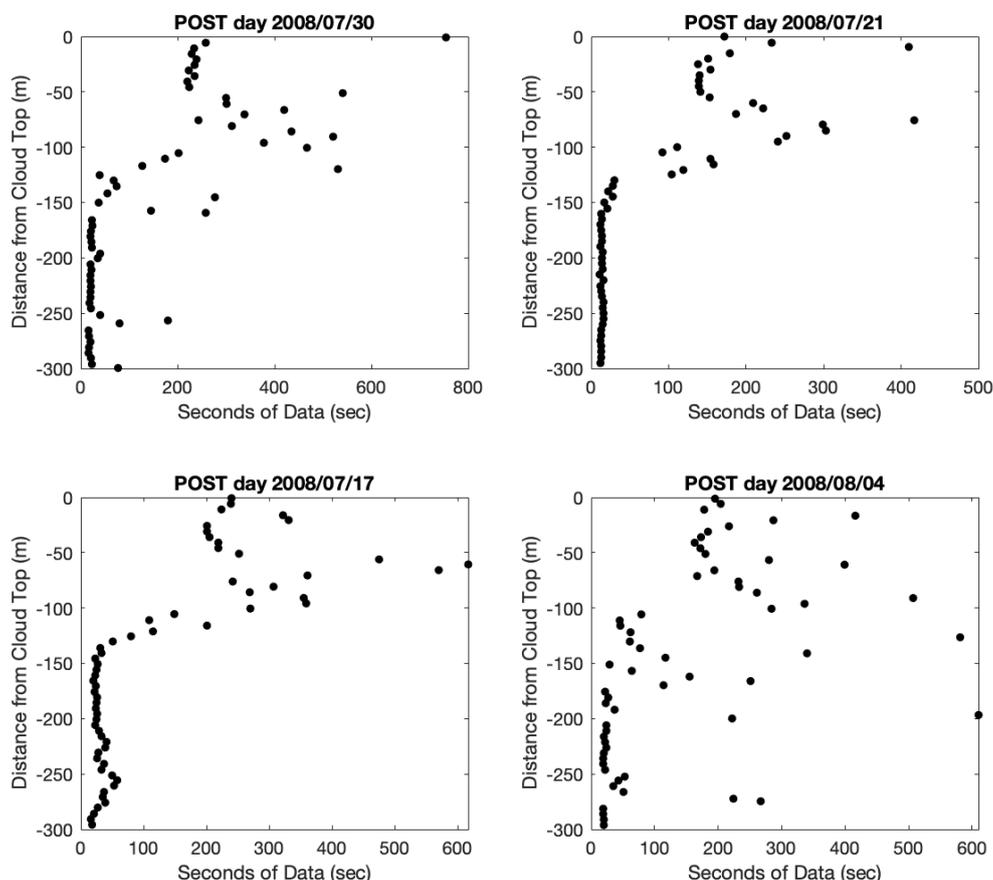


Figure 3. Amount of data for four of the five POST flights selected for additional analysis. Points represent seconds of non-zero data within each $5\text{ m } z_{shift}$ altitude bin and are concentrated within $\sim 150\text{ m}$ of cloud top.

base altitude as estimated from MODIS and aircraft (red and black dashed lines, respectively) also agree well. The alignment between this full set of MODIS and aircraft observations results in an accurate estimate of N_d as long as we interpret this value as a mid-cloud estimate. However, this day represents the only flight where this best case scenario occurs.

3.3.2 Case 2: POST day 2008/07/21

Unlike the strong agreement between MODIS and PDI seen in Case 1, Fig. 5 (POST day 2008/07/21) illustrates a day in which all of the profiles have very poor agreement. The MODIS retrieved number concentration is $N_d = 42 \pm 12\text{ cm}^{-3}$ while the in situ value is $N_d = 63 \pm 3\text{ cm}^{-3}$. This disagreement occurs despite the fact that the satellite cloud top effective radius (blue star) is well-matched to the PDI estimate ($13.0 \pm 0.9\text{ }\mu\text{m}$ vs. $13.0 \pm 0.5\text{ }\mu\text{m}$), indicating that cloud top r_e is not the source of the N_d disagreement.

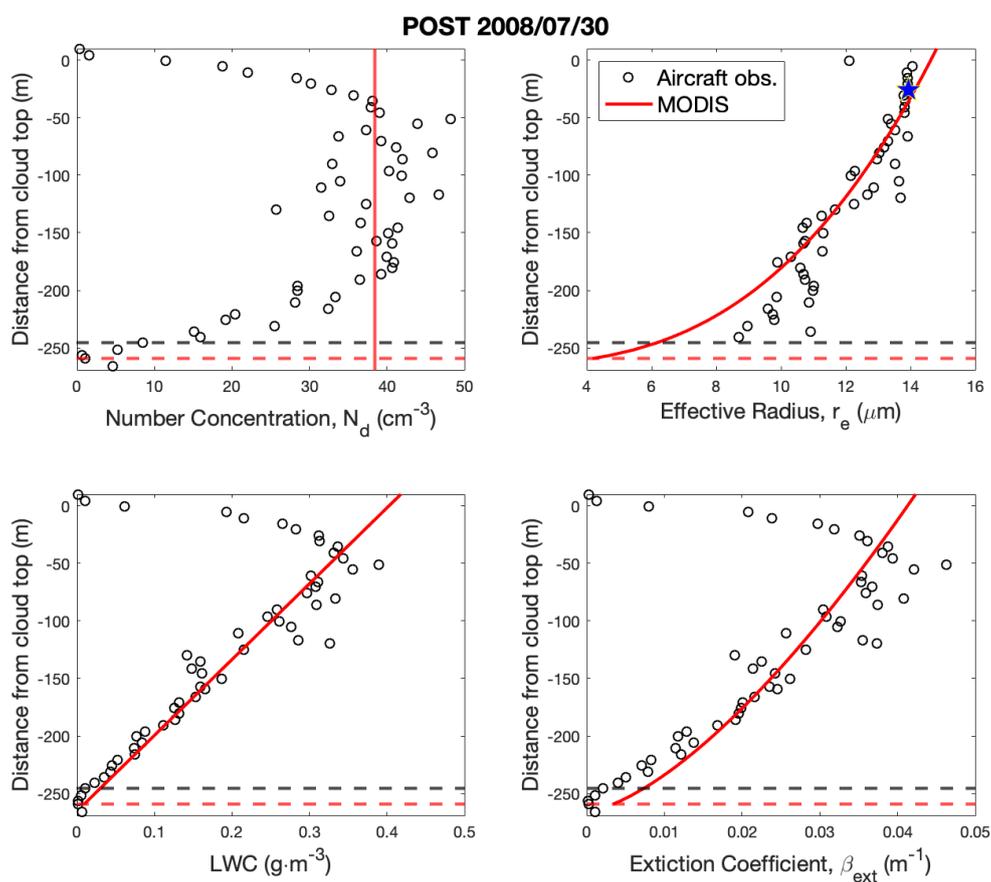


Figure 4. Profiles of N_d , r_e , L , and β for POST day 2008/07/30 comparing aircraft data (circles) and MODIS retrieval profiles (red lines). The aircraft measured cloud base is the black dashed line while the MODIS estimated cloud base is the red dashed line. Aircraft-derived r_e that most closely corresponds to the MODIS cloud top value is indicated by the blue star, and is placed at an altitude determined by the max weight from Eq. 2. MODIS retrieval profiles of N_d , r_e , L , and β are based on Eqs. 1, 9, 6, and 10, respectively.

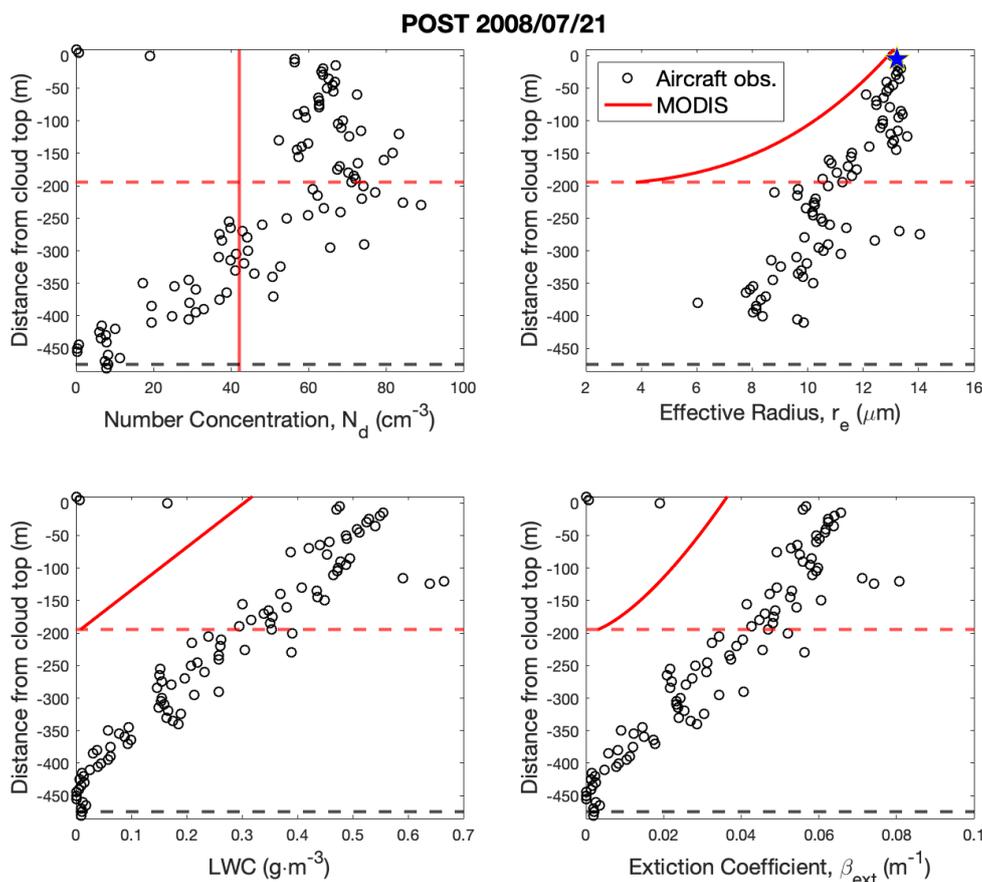


Figure 5. Comparison of profiles of N_d , r_e , L , and β from aircraft observations (circles) and MODIS retrievals (red lines) for POST on 2008/07/21. See Fig. 4 for more details.

While the MODIS and PDI r_e profiles agree at cloud top, at all other altitudes MODIS greatly underestimates effective radius. All else being equal, this should lead to an overestimation of N_d , which is not what we see. Instead, we attribute the MODIS N_d underestimation to disagreement with the MODIS τ_c retrieval. The MODIS $\beta(z)$ is determined by the MODIS retrieved τ_c while the MODIS $L(z)$ is heavily dependent on the MODIS retrieved τ_c (see Section 1.1). Consequently, any error in MODIS τ_c propagates to both the β and L profiles. Integration of the MODIS β profile results in a τ_c value smaller than observed, leading to an underestimation of N_d ($N_d \propto \tau_c^{1/2}$) as seen in Fig. 5.

3.3.3 Case 3: POST day 2008/07/17

During POST flight 2008/17/07, MODIS number concentration is $N_d = 260 \pm 31 \text{ cm}^{-3}$, almost a twice the PDI value of $N_d = 140 \pm 5 \text{ cm}^{-3}$ (Fig. 6). In contrast to the 2008/07/21 case, we attribute the cause of this disagreement to MODIS retrieved

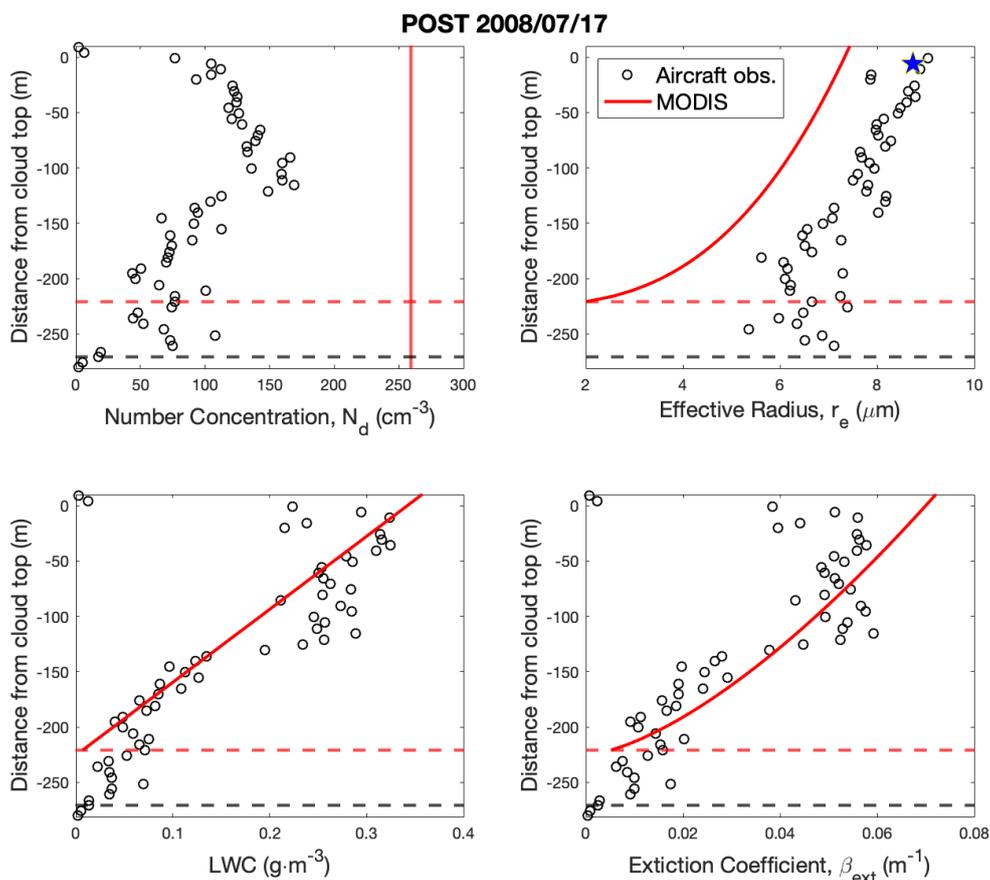


Figure 6. Comparison of profiles of N_d , r_e , L , and β from aircraft observations (circles) and MODIS retrievals (red lines) for POST on 2008/07/17. See Fig. 4 for more details.

cloud top r_e . The β and L profiles are in close agreement, presumably because MODIS and PDI τ_c are in agreement. However, both the PDI cloud top r_e as well as the PDI r_e vertical profile are greater than the MODIS estimates, leading to the large overestimation of N_d .

3.3.4 Case 4: POST day 2008/08/04

220 MODIS overestimates number concentration during POST flight 2008/08/04, finding an $N_d = 180 \pm 12 \text{ cm}^{-3}$ compared to a PDI value of $N_d = 122 \pm 5 \text{ cm}^{-3}$ (Fig. 7). More obviously, this case illustrates a day when the assumption that N_d is constant with altitude is not accurate, with the retrieval overestimating N_d at all altitudes. The r_e subplot shows that there is good agreement between PDI and MODIS in both the cloud top r_e value as well as the r_e vertical profile (in the region of most influence, i.e. within 50 m of cloud top). However, the MODIS β and L profiles differ to a substantial degree from PDI. The

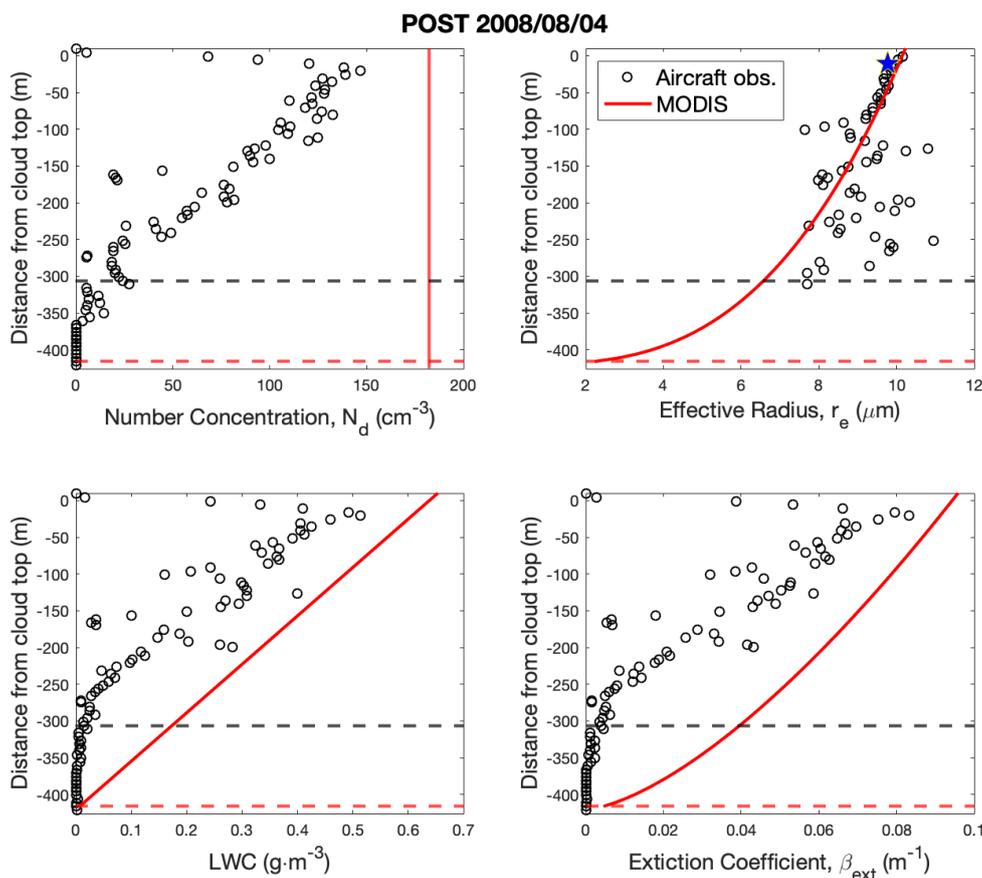


Figure 7. Comparison of profiles of N_d , r_e , L , and β from aircraft observations (circles) and MODIS retrievals (red lines) for POST on 2008/08/04. See Fig. 4 for more details.

225 MODIS β profile is greater than what is observed, meaning that the MODIS τ_c is also greater than the PDI value. Due to the relationship between τ_c and N_d , it follows that an overestimation in MODIS optical depth should lead to an overestimation in N_d which is indeed what we see in Fig. 7.

3.3.5 Case 5: POST day 2008/08/14

230 POST flight 2008/08/14 illustrates the degree of sensitivity in the MODIS N_d retrieval. Although the agreement between the profiles of r_e , L , and β is not perfect, it is reasonably close (Fig. 8). Despite this, MODIS significantly overestimates number concentration ($N_d = 98 \pm 5 \text{ cm}^{-3}$ compared to the PDI value of $N_d = 71 \pm 4 \text{ cm}^{-3}$). Even small errors in effective radius manifest as a large error in number concentration, presumably due to the non-linear relationship between the two. This suggests that satellite profile estimates must be fairly accurate in order to successfully retrieve N_d .

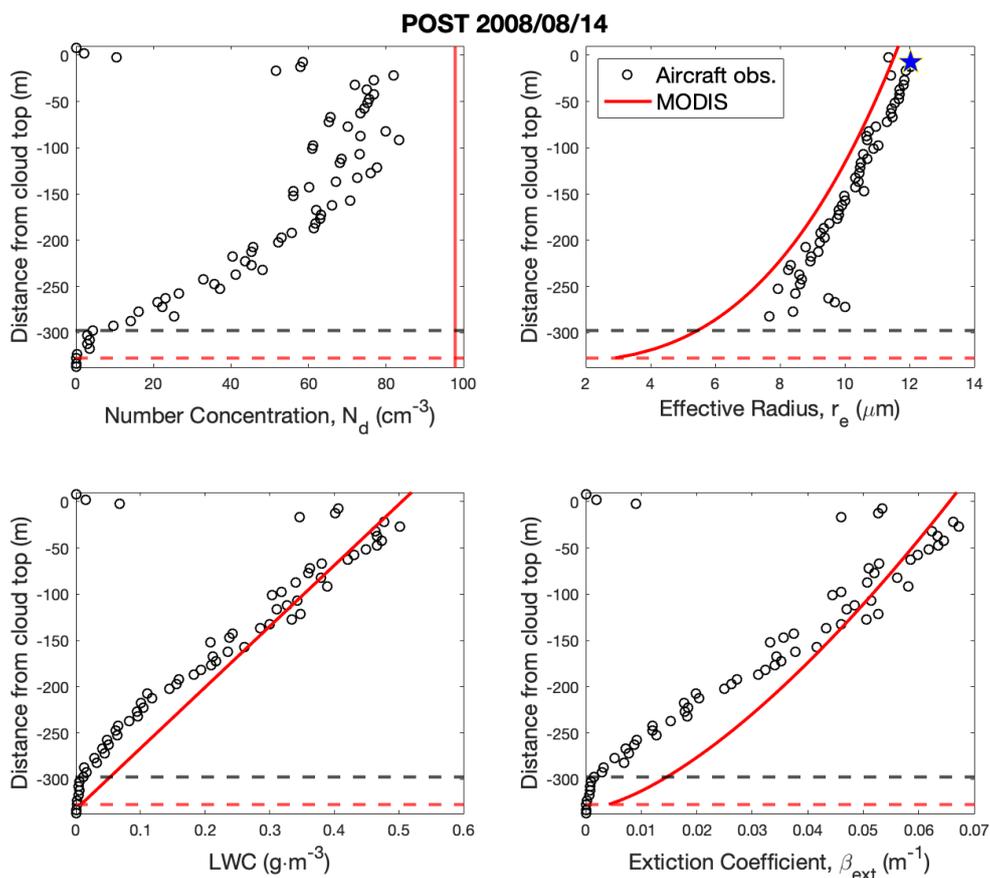


Figure 8. Comparison of profiles of N_d , r_e , L , and β from aircraft observations (circles) and MODIS retrievals (red lines) for POST on 2008/08/14. See Fig. 4 for more details.

3.3.6 Summary of POST Cases

235 Based on POST profile analysis of the important variables that determine MODIS number concentration, we find that there are cases in which all variables agree well with PDI observation as well as cases where one or more variables disagree. However, out of all eight days considered, there are no cases in which the MODIS N_d was correct due to compensating errors in the underlying variables (i.e. β , L , r_e). If the satellite number concentration is a match to the in situ value, it is due to correct estimations in all variables. Conversely, if even one variable disagrees with observation, N_d is also inaccurate. Fairly good
 240 agreement in all three profiles can also still yield significant discrepancy in N_d .

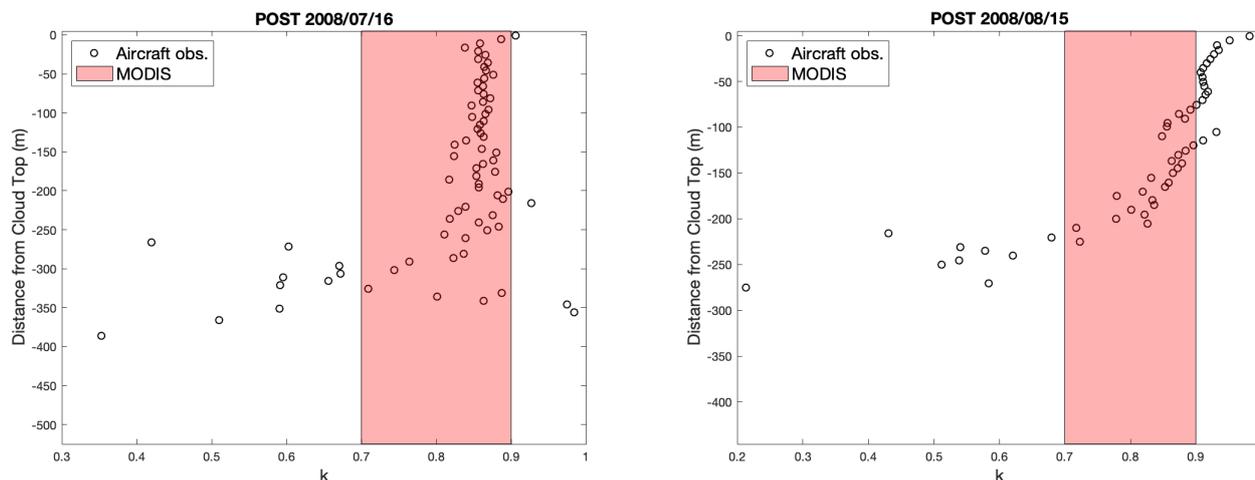


Figure 9. Comparison of k values from the POST campaign for days 2008/07/16 and 2008/08/15 with the MODIS assumed range of $0.7 \leq k \leq 0.9$ (red shading). The PDI values are calculated using Eq. 5 and represent flight averages at each 5 m z_{shift} bin.

3.4 Analysis of k and f_{ad}

The accuracy of the MODIS assumptions concerning f_{ad} and k can also affect retrieved number concentration. The MODIS retrieval assumes that f_{ad} is a constant equal to 0.6. If f_{ad} were significantly different from this assumed value, we would find that the assumed MODIS slope of $L(z)$ would disagree with observations. We find that for each of the 8 POST flights, the in situ $L(z)$ slopes are well matched to MODIS. This leads us to conclude that f_{ad} has little effect on the N_d retrieval for the cases that we analyzed.

MODIS assumes k is a constant of value 0.8. Analysis by Grosvenor et al. (2018) concludes that k ranges $0.7 \leq k \leq 0.9$ (see also Lebsock and Witte, 2023). We evaluate this conclusion using the POST data (two examples shown in Fig. 9). Through the bulk of cloud profiles, the MODIS assumption is found to be generally reasonable. Near cloud base, k can be much smaller. However, this can be considered inconsequential, as this region contributes little to the MODIS retrievals. There are also cases in which k exceeds this range near cloud top. However, occasional outlying behavior should not have a large impact on the retrieval and by default N_d as well.

3.5 Determination of Cloud Base

As illustrated by Fig. 5, the large inaccuracies in the MODIS profile assumptions for POST day 2008/07/21 are accompanied by a large satellite overestimation of cloud base altitude (over 200 m difference from observed). Fig. 10 compares MODIS derived cloud base to the observed cloud base altitude for all eight of the POST flights. MODIS is within 50 m of observation in 5 of the 8 cases. For the remaining three cases, MODIS cloud base differs by over 100 m, which is particularly notable since

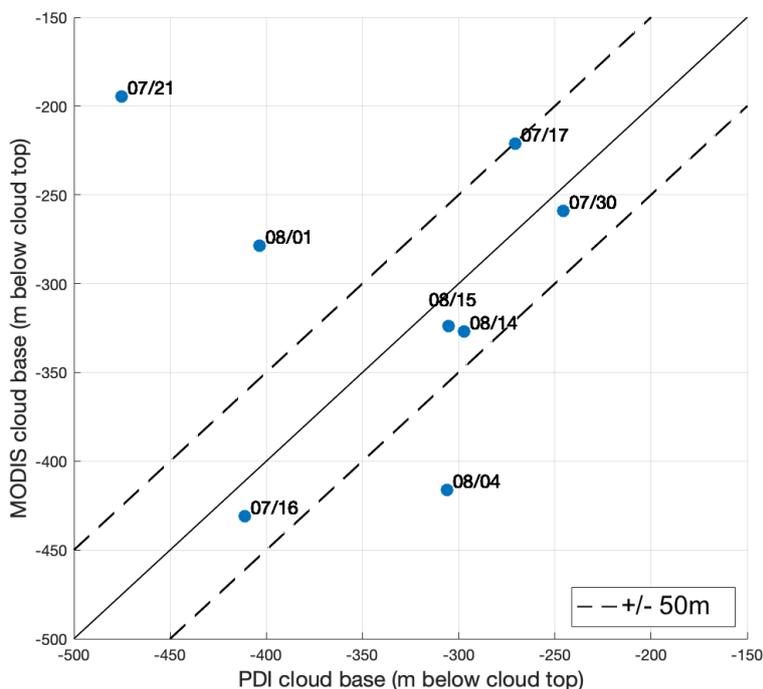


Figure 10. Comparison of MODIS and aircraft-estimated cloud base altitude for each day of the POST campaign. The 1:1 line (solid), and ± 50 m lines (dashed) are also shown.

this is a significant fraction of the cloud depth. Due to the small sample size, however, it is difficult to quantify the frequency of accurate MODIS estimations of cloud base.

260 4 Conclusions

In this study, we compare N_d derived from MODIS products with in situ observations recorded by the PDI instrument over three different field campaigns (MASE, VOCALS, and POST) sampling marine stratocumulus clouds. We also compare cloud microphysical and radiative variables relevant to the calculation of N_d using observations from the POST campaign. These variables include vertical profiles of r_e , L , and β .

265 Our results show that while there are instances in which MODIS predicts number concentration within sampling variability, there are a significant number of cases in which the satellite overestimates N_d . Our results suggest that the discrepancy between retrieved and in situ N_d can be $\pm 50\%$ or more, which is substantially larger than the $\pm 25\%$ that is proposed by Grosvenor et al. (2018). We find that the apparent overestimation bias in number concentration does not originate as a bias in MODIS r_e . This is consistent with the conclusions of Witte et al. (2018) which found no obvious bias between MODIS and PDI derived



270 effective radius. We also find that it is possible for N_d to be accurately retrieved with a poor retrieval of r_e , presumably due to compensating errors in other retrieval parameters.

Based on analysis of vertical cloud profiles of r_e , L , and β from the POST campaign, we do not attribute N_d discrepancy to a single error source. Instead, we show that there are several different cases which result in incorrect number concentration:

1. MODIS incorrectly predicts profiles of r_e , L , and β
- 275 2. MODIS incorrectly predicts profiles of L and β but accurately estimates r_e (either the full r_e profile or at the most influential altitude near cloud top)
3. MODIS incorrectly predicts profiles of r_e but accurately estimates L and β profiles

Accurate N_d retrievals are representative of mid-cloud values. Although MODIS assumes number concentration is constant through a cloud's vertical profile, even in the best case scenarios it appears to be a poor reflection of cloud top and cloud base
280 conditions. We also show one case where the assumption that N_d is constant with altitude is not accurate.

In order to improve the MODIS N_d retrieval, it would be beneficial to acquire more data collected in a manner similar to the POST campaign, i.e., with repeated sawtooth-like penetrations through cloud top. Multiple profiles near cloud top allow for deeper analysis of the underlying variables in the retrieval which can be used to more accurately quantify sources of error.

Data availability. PDI data from POST and VOCALS are freely available at <https://data.eol.ucar.edu>, and from MASE at <https://doi.org/10.5281/zenodo.1035928>. MODIS Level 2 retrievals are available from <https://search.earthdata.nasa.gov>.
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Author contributions. SRP performed the data analysis and wrote the manuscript. MKW and PYC designed the study concept, obtained data, and edited the manuscript.

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References

- Carman, J. K., Rossiter, D. L., Khelif, D., Jonsson, H. H., Faloon, I. C., and Chuang, P. Y.: Observational constraints on entrainment and the entrainment interface layer in stratocumulus, *Atmos. Chem. Phys.*, 12, 11 135–11 152, <https://doi.org/10.5194/acp-12-11135-2012>, 2012.
- Chuang, P. Y., Saw, E. W., Small, J. D., Shaw, R. A., Sipperley, C. M., Payne, G. A., and Bachalo, W. D.: Airborne phase doppler interferometry for cloud microphysical measurements, *Aerosol Sci. Tech.*, 42, 685–703, <https://doi.org/10.1080/02786820802232956>, 2008.
- 295 Gerber, H., Frick, G., Malinowski, S. P., Jonsson, H., Khelif, D., and Krueger, S. K.: Entrainment rates and microphysics in POST stratocumulus, *J. Geophys. Res. Atmos.*, 118, 12,094–12,109, <https://doi.org/https://doi.org/10.1002/jgrd.50878>, 2013.
- Grosvenor, D. P., Sourdeval, O., Zuidema, P., Ackerman, A., Alexandrov, M. D., Bennartz, R., Boers, R., Cairns, B., Chiu, J. C., Christensen, M., Deneke, H., Diamond, M., Feingold, G., Fridlind, A., Hunerbein, A., Knist, C., Kollias, P., Marshak, A., McCoy, D., Merk, D., Painemal, D., Rausch, J., Rosenfeld, D., Russchenberg, H., Seifert, P., Sinclair, K., Stier, P., van Diedenhoven, B., Wendisch, M., Werner, F., Wood, R., Zhang, Z., and Quaas, J.: Remote sensing of droplet number concentration in warm clouds: A review of the current state of knowledge and perspectives, *Rev. Geophys.*, 56, 409–453, <https://doi.org/https://doi.org/10.1029/2017RG000593>, 2018.
- Lebsock, M. D. and Witte, M.: Quantifying the dependence of drop spectrum width on cloud drop number concentration for cloud remote sensing, *Atmos. Chem. Phys.*, 23, 14 293–14 305, <https://doi.org/10.5194/acp-23-14293-2023>, 2023.
- 305 Lu, M.-L., Conant, W. C., Jonsson, H. H., Varutbangkul, V., Flagan, R. C., and Seinfeld, J. H.: The Marine Stratus/Stratocumulus Experiment (MASE): Aerosol-cloud relationships in marine stratocumulus, *J. Geophys. Res. Atmos.*, 112, <https://doi.org/https://doi.org/10.1029/2006JD007985>, 2007.
- Mechoso, C. R., Wood, R., Weller, R., Bretherton, C. S., Clarke, A. D., Coe, H., Fairall, C., Farrar, J. T., Feingold, G., Garreaud, R., Grados, C., McWilliams, J., Szoek, S. P. d., Yuter, S. E., and Zuidema, P.: Ocean Cloud Atmosphere Land Interactions in the Southeastern Pacific: The VOCALS Program, *Bull. Amer. Meteorol. Soc.*, 95, 357–375, <https://doi.org/10.1175/BAMS-D-11-00246.1>, 2014.
- 310 Miles, N. L., Verlinde, J., and Clothiaux, E. E.: Cloud droplet size distributions in low-level stratiform clouds, *J. Atmos. Sci.*, 57, 295–311, [https://doi.org/10.1175/1520-0469\(2000\)](https://doi.org/10.1175/1520-0469(2000)), 2000.
- Min, Q., Joseph, E., Lin, Y., Min, L., Yin, B., Daum, P. H., Kleinman, L. I., Wang, J., and Lee, Y. N.: Comparison of MODIS cloud microphysical properties with in-situ measurements over the Southeast Pacific, *Atmos. Chem. Phys.*, 12, 11 261–11 273, <https://doi.org/10.5194/acp-12-11261-2012>, 2012.
- 315 Noble, S. R. and Hudson, J. G.: MODIS comparisons with northeastern Pacific in situ stratocumulus microphysics, *J. Geophys. Res. Atmos.*, 120, 8332–8344, <https://doi.org/https://doi.org/10.1002/2014JD022785>, 2015.
- Painemal, D. and Zuidema, P.: Assessment of MODIS cloud effective radius and optical thickness retrievals over the Southeast Pacific with VOCALS-REx in situ measurements, *J. Geophys. Res. Atmos.*, 116, D24 206, <https://doi.org/10.1029/2011JD016155>, 2011.
- 320 Platnick, S.: Vertical photon transport in cloud remote sensing problems, *J. Geophys. Res. Atmos.*, 105, 22 919–22 935, <https://doi.org/10.1029/2000JD900333>, 2000.
- Witte, M. K., Yuan, T., Chuang, P. Y., Platnick, S., Meyer, K. G., Wind, G., and Jonsson, H. H.: MODIS retrievals of cloud effective radius in marine stratocumulus exhibit no significant bias, *Geophys. Res. Lett.*, 45, 10,656–10,664, <https://doi.org/https://doi.org/10.1029/2018GL079325>, 2018.
- 325 Zheng, X., Albrecht, B., Jonsson, H. H., Khelif, D., Feingold, G., Minnis, P., Ayers, K., Chuang, P., Donaher, S., Rossiter, D., Ghate, V., Ruiz-Plancarte, J., and Sun-Mack, S.: Observations of the boundary layer, cloud, and aerosol variability in the southeast Pacific near-coastal marine stratocumulus during VOCALS-REx, *Atmos. Chem. Phys.*, 11, 9943–9959, <https://doi.org/10.5194/acp-11-9943-2011>, 2011.